Date 11/9/23

# Syllabus

- Thermal Phy.
- Polarization of light
- Dielectrics
- Magnetic Properties of matter.
- Vector Analysis
- E.M Waves.

Syllabus by N.P

- Vector analysis
- Polarization of light
- Electromagnetic theory
- Modern Phy.
- Quantum mechanics
- Theory of relativity (Spectrum theory of relativity)
- LASER Phy.

# Special Theory of Relativity

# Frame of reference :- It is just a system which specifies the position & momentum of two dimensional object.

or

A coordinate system w.r.t. which we measure the position of a particle is known a frame of reference.

## Frame of reference.

- Inertial frame of reference. or Unaccelerated frame of reference.
- Non-Inertial frame of reference. or Accelerated frame of reference.

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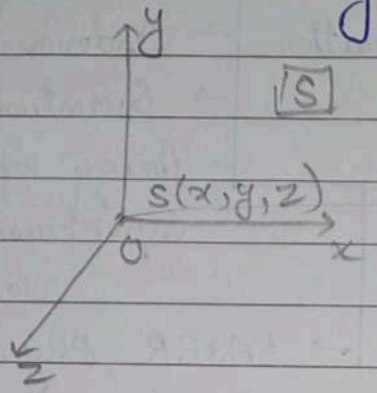
or

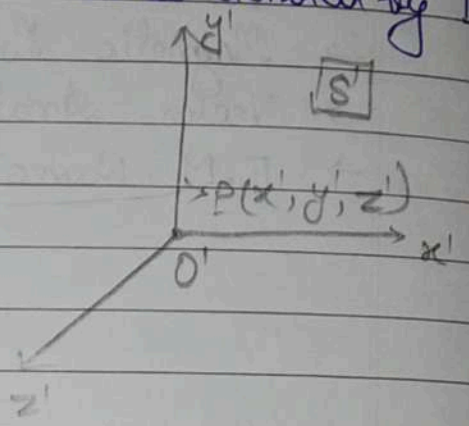
Stationary frame of reference.

- It is denoted by [S]




Moving frame of reference.

- It is denoted by [S']




★ Inertial frame of reference :-

It is a frame of reference in which Newton's first law of motion holds.

Or

An inertial frame of reference is a frame in which Newton's laws of motion work. A frame of reference that moves with constant velocity w.r.t. an inertial frame is also an inertial frame.

★ Non- Inertial frame of reference :-

It is a frame of reference that undergoes acceleration w.r.t. an inertial frame. An accelerometer at rest in a non-inertial frame will, in general, detect a non-zero acceleration.

★ Ether medium :-

It is a theoretical universal substance believed during the 19th century to act as the medium for transmission of electromagnetic waves. (Eg light & X-rays)

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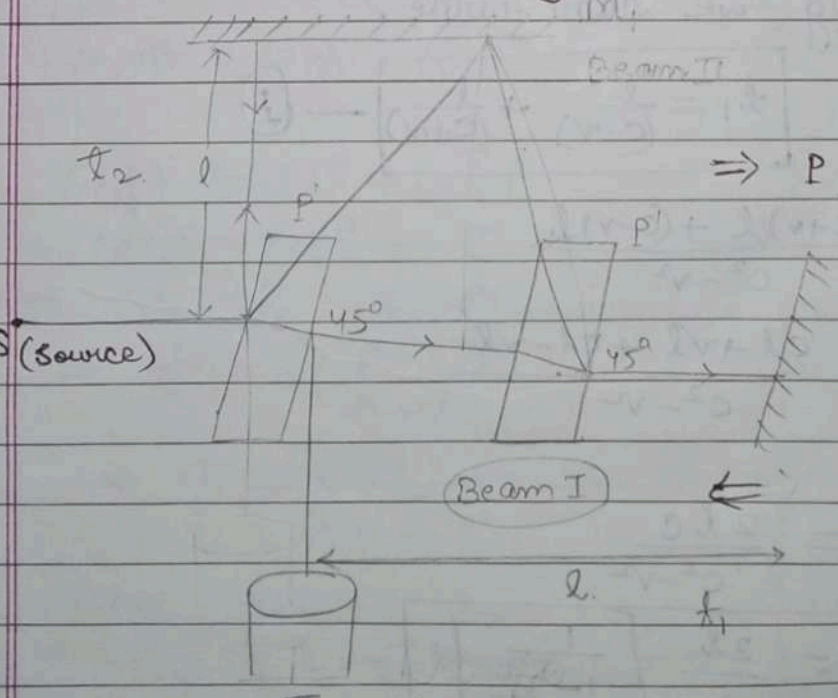
# Michelson Morley Experiment :- (M.M. Exp.) ⇒

In 19th century, scientists have assumed that in hypothetical medium called ether medium, is required for the propagation of the light.

It was considered that the ether exists uniformly in the space & it is at rest relative to the earth & other planets.

The basic purpose of M.M. experiments was to confirm the existence of stationary ether. The existence of stationary ether can be confirmed if we can measure the absolute velocity of earth w.r.t. the stationary ether.

⇒ Experimental arrangement :-




⇒ P to $M_2$ is (C-v)

⇐ $M_2$ to P is (C+v).

Note!!

$PM_1 = PM_2 = l$.

P & P' are partially silvered glass plate.

$M_1$ & $M_2$ ~~are~~ = Interference of light

The main components of M.M experiments are mirrors $M_1$ & $M_2$, partially silvered glass plate, P & P' of same size & a source of light (S) & a telescope T.

The light beam moves to mirror $M_1$ & the other moves to mirror $M_2$. These two mirrors beams are reflected back by these mirrors & again are recombined at plate P. Finally, they enter the telescope & produce interference.

According to Galilean Transformation, the velocity of light in a frame moving with constant velocity 'v' relative to stationary ether, ether from P to $M_2$ is $(c-v)$ while from $M_2$ to P is $(c+v)$.

If $t_1$ is the time taken by transmitted 'v' from P to $M_2$ and back.

From fig. we can write,

$$\boxed{t_1 = \frac{l}{(c-v)} + \frac{l}{(c+v)}} \quad \text{---} (1)$$

$$\Rightarrow t_1 = \frac{(c+v)l + (c-v)l}{c^2 - v^2}$$

$$\Rightarrow t_1 = \frac{cl + vl + cl - vl}{c^2 - v^2}$$

$$\Rightarrow t_1 = \frac{2lc}{c^2 - v^2}$$

$$\Rightarrow \boxed{t_1 = \frac{2l}{c}\left[\frac{1}{1 - \frac{v^2}{c^2}}\right]} \quad \text{Ans}$$

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Q: $t_1 = \frac{2l}{c}\left[1 - \frac{v^2}{c^2}\right]^{-1}$, solve with the help of binomial expansion.

By using binomial expansion theorem and neglecting the higher order terms, we get

$$t_1 = \frac{2l}{c}\left[1 + \frac{v^2}{c^2} \cdots\right] \quad \text{---} \quad (2)$$

Now, the distance travelled by the light beam of in time '$t_1$' is given by

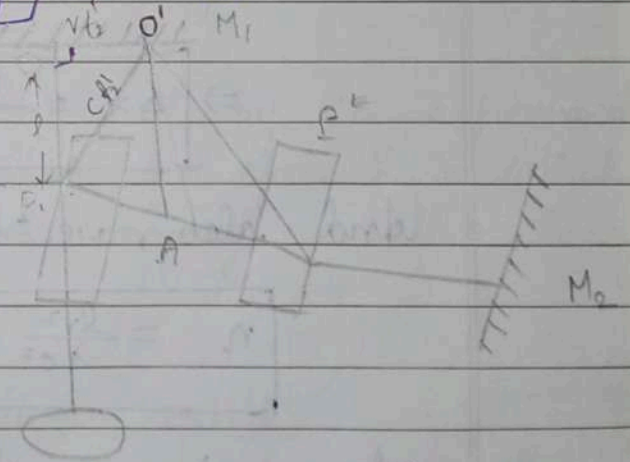
$$x_1 = t_1 \cdot c.$$

$$\Rightarrow \boxed{x_1 = 2l\left[1 + \frac{v^2}{c^2} \cdots\right]} \quad \text{---} \quad (3)$$

From figure ;-

$\boxed{PO = l,\ O'P = ct_2',\ OO' = vt_2'}$

$(PO')^2 = (PO)^2 + (OO')^2$




$$(ct_2')^2 = (l)^2 + (vt_2')^2$$

$$\Rightarrow (ct_2')^2 - (vt_2')^2 = l^2$$

$$\Rightarrow (c^2 - v^2)t_2'^2 = l^2$$

$$\Rightarrow t_2' = \left(\frac{l^2}{c^2 - v^2}\right)^{1/2}.$$

$$\Rightarrow t_2' = \frac{l}{c}\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{---} \quad (4)$$

Total time,

$$t_2 = 2t_2'$$

$$t_2 = \frac{2l}{c}\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow t_2 = \frac{2l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

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By using binomial theorem, and higher order terms we get

$$t_2 = \frac{2l}{c}\left[1 + \frac{v^2}{2c^2} \cdots\right] \quad \text{—— (5)}$$

$$x_2 = ct_2$$

$$\Rightarrow x_2 = 2l\left[1 + \frac{v^2}{2c^2} \cdots\right] \quad \text{—— (6)}$$

$$\boxed{\begin{matrix} x_1 = x_2 \\ t_1 \neq t_2 \end{matrix}}$$

With the help of equ (iii) & (vi), we get the path difference ($\Delta x$).

$$\Delta x = x_1 - x_2$$

$$\Rightarrow \Delta x = 2l\left(1 + \frac{v^2}{c^2}\right) - 2l\left(1 + \frac{v^2}{2c^2}\right)$$

$$\Rightarrow \Delta x = 2l\left[1 + \frac{v^2}{c^2} - 1 - \frac{v^2}{2c^2}\right]$$

$$\Rightarrow \Delta x = \frac{2lv^2}{2c^2}$$

$$\Rightarrow \boxed{\Delta x = \frac{lv^2}{c^2}}$$

And also, we know that, $\Delta = n\lambda$ & $\boxed{n = \frac{\Delta}{\lambda}}$

$$\boxed{n = \frac{lv^2}{\lambda c^2}} = \boxed{\text{Interference pattern}}$$

where, $\lambda$ = wavelength of light used

If the apparatus is rotated through 90°, the reflected & transmitted beam get interchanged & the path difference of $(lv^2/c^2)$ will be produced in opposite direction.

This way, the total path difference b/w the interference beams became $\left(\frac{2lv^2}{\lambda c^2}\right)$ & the interference

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pattern would be shifted as $\boxed{n = \frac{2lv^2}{\lambda c^2}}$ —— (7)

In the above equo (7), we have to put

$l = 11\,m$, $v = 3 \times 10^4\ m/s$, $c = 3 \times 10^8\ m/s$.

$\lambda = 5.5 \times 10^{-7}\ nm$, ~~$n = 2$~~

$\boxed{n = \frac{2lv^2}{\lambda c^2}}$ $\boxed{n = 0.4}$ (X) | $n = 0.004$ ✓

This displacement of fringe width would have been observed easily, since the apparatus used in this experiment was capable of observing a fringe shift as small as 0.01. However, experimentally, no significant fringe shift would be observed.

This negative result observed by the experiment suggest that the medium or space in which light propagates, is not moving relative to the Earth

# Explanation of negative result

Lorentz Fitzgerald Contraction Hypothesis :-

As per this hypothesis, all bodies moving with velocity 'v' are contracted in the dirⁿ of motion by a factor $\sqrt{1 - \frac{v^2}{c^2}}$.

If '$l_0$' be the length of a body at rest w.r.t. ether and '$l$' be its length, then

$$l = l_0\sqrt{1-\frac{v^2}{c^2}}$$

Therefore, $t_1 = \frac{2l}{c}\left(1+\frac{v^2}{c^2}\right)$.

$$\Rightarrow t_1 = \frac{2l_0\sqrt{1-\frac{v^2}{c^2}}}{c}\left(1+\frac{v^2}{c^2}\right).$$

$$\Rightarrow t_1 = \frac{2l_0}{c}\left(1-\frac{v^2}{c^2}\right)^{1/2}\left(1+\frac{v^2}{c^2}\right).$$

$$\Rightarrow t_1 = \frac{2l_0}{c}\left\{1+\frac{v^2}{c^2}-\frac{v^2}{2c^2}-\frac{v^4}{2c^4}+\cdots\right\}.$$

$$\Rightarrow t_1 = \frac{2l_0}{c}\left(1+\frac{v^2}{2c^2}\right)$$

& $t_2 = \frac{2l_0}{c}\left(1+\frac{v^2}{2c^2}\right)$

$$\therefore \boxed{t_1 = t_2 = \frac{2l_0}{c}\left(1+\frac{v^2}{2c^2}\right)}$$

It means the time taken by reflected and transmitted beams are same. So, no shift fringe is observed. These shows that (-ve) result of M. M experiment.

# Lorentz Transformation

$$x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} \longrightarrow (a)$$

$$y' = y \longrightarrow (b)$$

$$z' = z \longrightarrow (c)$$

$$t' = \frac{t-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \longrightarrow (d)$$

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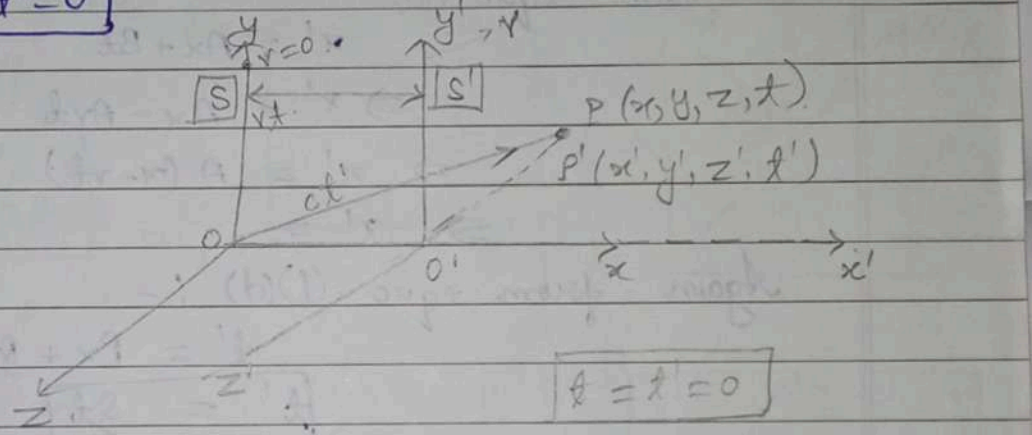
Since, Galilean transformation do not satisfies the postulates of that the speed of light is an universal constant. Einstein propose an alternative method for describing an the same event in different inertial frame system.

Let $S(x, y, z, t)$ is an stationary frame & $S'(x', y', z', t')$ is a moving frame moving with the constant velocity 'v' & let the origin of two reference frame coincide at

$$\boxed{t = t' = 0}$$




Let us Consider the most general transformation equo

$$\boxed{\begin{aligned} x' &= Ax + Bt \longrightarrow (a) \\ y' &= y \longrightarrow (b) \\ z' &= z \longrightarrow (c) \\ t' &= Px + Qt \longrightarrow (d) \end{aligned}} \longrightarrow (i)$$

⇒ Determination of constant A, B, P & Q :-

Let us consider the observer is in S' frame. Stationary frame ⇒ $S(x, y, z, t)$ see in the origin of moving frame $S'(x', y', z', t')$. In this case

$$\boxed{x = vt}$$
$$\boxed{x' = 0}$$

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$x' = Ax + Bt$.

$0 = Avt + Bt$.

$Bt = -Avt$.

$\boxed{B = -Av}$ ⟶ (2)

Again, the observer is in moving frame & see in the origin of stationary frame.

$x = 0$.

~~$x' = vt'$~~.

$\boxed{x' = -vt'}$.

From equo (i),

$x' = Ax + Bt$.

$\Rightarrow x' = Ax - Avt$.

$\Rightarrow x' = A(x - vt)$ $\Rightarrow \boxed{x' = -Avt}$ ⟶ (A)

$\Rightarrow x' =$

Again, from equo (1)(d) :-

$t' = Px + Qt$.

$\boxed{t' = Qt}$.

Again, $\boxed{x' = -vt'}$ $\Rightarrow \boxed{x' = -vQt}$ ⟶ (B)

From equo (A) & (B), $\boxed{A = Q}$ ⟶ (3)

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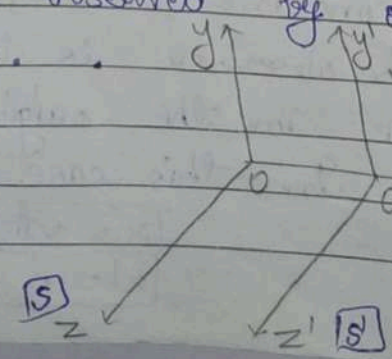
A/C 2nd theory of postulate

According to special theory of relativity the speed of light is constant. So, the velocity of light pulse which spreads out from the common origin observed by observer 'O' & 'O' should be the same.

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S  S'

$x = ct$

$x' = ct'$

$x' = ct'$

$\Rightarrow Ax + Bt = c(Px + Qt)$

$\Rightarrow Ax - Avt = c(Pct + At)$

$\Rightarrow A(x - vt) = c^2Pt + Act$

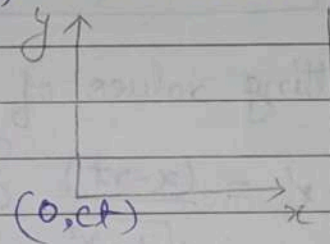
$\Rightarrow -Av = c^2P$

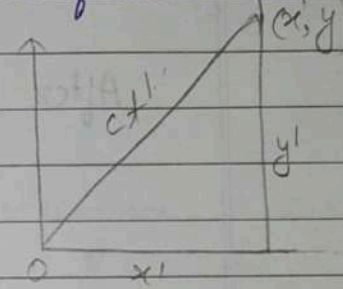
$\Rightarrow \boxed{P = \frac{-Av}{c^2}} \longrightarrow (4)$

Consider the light pulse is emitted about x-axis. In this case, coordinate in S-frame is given by

$x = 0$

$y = ct$




from equ fig.

$\boxed{x'^2 + y'^2 = c^2t'^2}$

From equ (A),

$\Rightarrow x' = A(x - vt)$

from (i) $y' = y$

$t' = A\left(\frac{-vx}{c^2} + t\right)$

& $x = 0$

$y = ct$

~~& $t' = Px + Qt$~~

~~$t' = -A$~~

$A = ?, \quad B = ?, \quad P = ?, \quad Q = ?$

$\Rightarrow$ Eqn:- $\because x'^2 + y'^2 = c^2t'^2$

$\Rightarrow A^2(x - vt)^2 + y^2 = c^2A^2\left(\frac{+vx}{c^2} + t\right)^2$

$\Rightarrow A^2(0 - vt)^2 + c^2t^2 = c^2A^2\left(\frac{-vx_0}{c^2} + t\right)^2$

$\Rightarrow A^2v^2t^2 + c^2t^2 = c^2A^2t^2$

$\Rightarrow A^2v^2t^2 - c^2A^2t^2 = -c^2t^2$

$\Rightarrow A^2(v^2 - c^2) = -c^2 \Rightarrow A = \sqrt{\frac{-c^2}{v^2 - c^2}}$

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$$A = \sqrt{\frac{c^2}{c^2 - v^2}} \Rightarrow \boxed{A = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$B = -Av = \frac{-v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$P = \frac{-Av}{c^2} = \frac{-v^2}{c^2\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\therefore \boxed{Q = A = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

After putting values of A, B, P & Q in (1), we get.

$$x' = \frac{(x - vt)}{\sqrt{1 - \frac{v^2}{c^2}}} \longrightarrow (6)$$

$$y' = y$$

$$z' = z.$$

$$\boxed{t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

Equo (6) is known as Lorentz transformation equo. Similarly, we have to find Inverse Lorentz transformation equo, and inverse lorentz equo is

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + vx'/c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \longrightarrow (7)$$

Equo (7) is known as (inverse lorentz Equation.)

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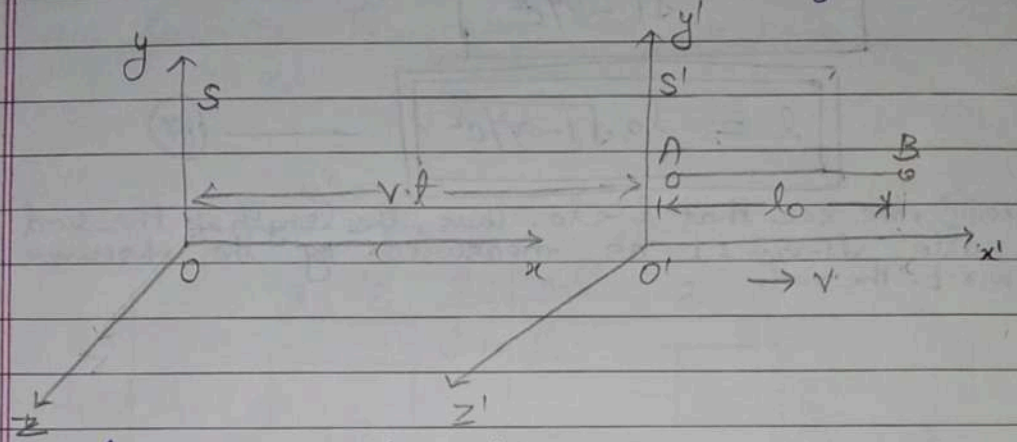
# Length Contraction :-

⇒ Application of Lorentz transformation:

# Lorentz Length Contraction :- Moving objects are shorter in the direction of motion.




Let us consider two frame of reference S & S' with their x-axis coinciding at $t = 0$ & a rod AB at rest in [S'] frame.

Let $x_1'$ & $x_2'$ = co-ordinates of the ends of the rod at any instant in S' frame.

$x_1$ & $x_2$ → Coordinates of the ends of the rod at the for same instance in S-frame.

$x_2' - x_1' = l_0$ = proper length

$x_2 - x_1$ = length of the rod measured in observer's frame of reference, 'S'.

According to Lorentz Transformation equ

$$x_2' = \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \longrightarrow (i)$$

$$x_1' = \frac{x_1 - vt}{\sqrt{1 - v^2/c^2}} \longrightarrow (ii)$$

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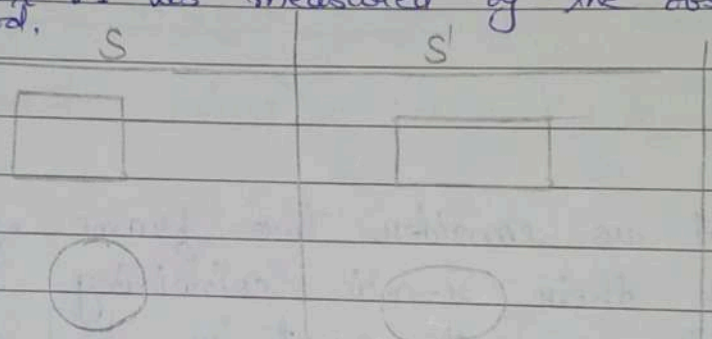
Subtracting equ (ii) from equ (i),

$$x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1 - v^2/c^2}}$$

$$l_0 = \frac{l}{\sqrt{1 - v^2/c^2}}$$

$$l = l_0\sqrt{1 - v^2/c^2} \quad \text{——— (iii)}$$

From Equ (iii), we see that $l < l_0$. Thus, the length of the rod is reduced in the ratio $\sqrt{1-v^2/c^2} : 1$ as measured by the observer moving with velocity $v$ w.r.t. the rod.




Q: Explain / derive Lorentz Transformation Equ.

Q A rod 1 m in length located along the x'-axis moves with the speed 0.75 c w.r.t. a stationary observer. What is the length of the rod as measured by the stationary observer?

$l_0 = 1\text{m}$, $l = ?$, $v = 0.75c$

$l_0 = l_0\sqrt{1 - v^2/c^2}$

$\Rightarrow l = 1\sqrt{1 - (0.75c)^2/c^2}$

$= 1\sqrt{1 - \frac{75^2 c^2}{100^2 c^2}}$

$= \sqrt{\frac{100^2 - 75^2}{100 \times 100}}$

$= \sqrt{\frac{10000 - 5625}{10000}}$

$= \frac{4375}{10000}$

$= 0.04375$ m

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Q. Show that $x^2+y^2+z^2-c^2t^2$ is invariant under Lorentz transformation.

⇒ Soln:- Let $x^2+y^2+z^2-c^2t^2 = s$ (given in invariant) ——— (i)

$R.H.S. = x'^2+y'^2+z'^2-c^2t'^2 = s'$

A/c Lorentz Transformation, we have,

$\because x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}$.

$y' = y$

$z' = z$

$t' = \frac{t-vx/c^2}{\sqrt{1-\frac{v^2}{c^2}}}$.

$\therefore \frac{(x-vt)^2}{\left(1-\frac{v^2}{c^2}\right)} + y^2 + z^2 - c^2\cdot\frac{\left(t-\frac{vx}{c^2}\right)^2}{1-\frac{v^2}{c^2}}$

$= \frac{x^2+v^2t^2-2xvt}{\frac{c^2-v^2}{c^2}} + y^2 + z^2 - c^2\frac{\left(t^2+\frac{v^2x^2}{c^4}-\frac{2txv}{c^2}\right)}{\frac{c^2-v^2}{c^2}}$.

$= x^2+v^2t^2-2xvt + c^2y^2-$

$= \frac{1}{1-\frac{v^2}{c^2}}\left[x^2+v^2t^2-2xvt - c^2\left(t^2+\frac{x^2v^2}{c^4}-\frac{2txv}{c^2}\right)\right]+y^2+z^2$

$= \frac{1}{1-\frac{v^2}{c^2}}\left[x^2\left(1-\frac{v^2}{c^2}\right)-c^2t^2+v^2t^2\right]+y^2+z^2$

$= \frac{1}{1-\frac{v^2}{c^2}}\left[x^2\left(1-\frac{v^2}{c^2}\right)-c^2t^2\left(1-\frac{v^2}{c^2}\right)\right]+y^2+z^2$

$= \frac{\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{v^2}{c^2}\right)}\left[x^2-c^2t^2\right]+y^2+z^2$

$= x^2+y^2+z^2-c^2t^2$

$\therefore x'^2+y'^2+z'^2-c^2t'^2 = x^2+y^2+z^2-c^2t^2$.

This shows that the quantity $x^2+y^2+z^2-c^2t^2$ is same in both frame of references. So, this is invariant under Lorentz Transformation. Equ. 4

Q. How fast would a rocket to go relative to a observer for its length to be contracted to 99% of its length at rest?

⇒ Given :- $\boxed{l = l_0 \times \frac{99}{100}}$

$\because l < l_0$

$\because l = l_0\sqrt{1 - \frac{v^2}{c^2}}$

$\Rightarrow l_0 \times \frac{99}{100} = l_0\sqrt{1 - \frac{v^2}{c^2}}$

$\Rightarrow \left(\frac{99}{100}\right)^2 = 1 - \frac{v^2}{c^2}$

$\Rightarrow \frac{99 \times 99}{100 \times 100} = 1 - \frac{v^2}{3 \times 3 \times 10^{16}}$

$\Rightarrow \frac{(99)^2}{100^2} \times 9 \times 10^{16} = 9 \times 10^{16} - v^2$

$\Rightarrow (99)^2 \times 9 \times 10^{12} = 9 \times 10^{16} - v^2$

$\Rightarrow 9801 \times 9 \times 10^{12} = 90000 \times 10^{12} - v^2$

$\Rightarrow 88209 \times 10^{12} = 90000 \times 10^{12} - v^2$

$\Rightarrow v^2 = \sqrt{90,000 - 88209} \times 10^6$

$\Rightarrow v = \sqrt{1791} \times 10^6$

$\Rightarrow v = 42.3 \times 10^6 \text{ m/s}$ Ans

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# Time Dilation

[Moving clock runs slower than stationary clock]
The word dilation means to lengthen.

The time interval is not absolute but it depends on the relative motion b/w the event & the observer. Let us suppose that, there are two clocks exactly synchronise initially, i.e., they are recording the same time interval at rest. Further 1 clock be placed in stationary frame of reference and the other be placed in moving frame of reference.

$(t_2' - t_1' = \Delta t) \Rightarrow$ Time interval by the moving clock.

$t_2 - t_1 = \Delta t_0 \Rightarrow$ Time interval by stationary clock.

According to Lorentz transformation equ,

$$t_1' = \frac{t_1 - vx/c^2}{\sqrt{1 - v^2/c^2}} \quad \text{— (i)}$$

$$t_2' = \frac{t_2 - vx/c^2}{\sqrt{1 - v^2/c^2}} \quad \text{— (ii)}$$

Subtracting equ (ii) from (i),

$$t_2' - t_1' = \frac{t_2 - t_1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{— (iii)}$$

$$\Rightarrow \Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} \quad \text{— (iv)}$$

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$\Delta t > \Delta t_0$ i.e,

From equ (i), it is clear that, time interval, $\Delta t$, as it measured by moving clock is more than the time interval, $\Delta t_0$ as measured by stationary clock. It means that the moving clock is slowed down. This phenomenon is called time dilation.

# Velocity Addition :-

Let us consider that a particle is moving relative to both (S) & (S') frame & an observer in S measures its 3 velocity components to be

$$u_x = \frac{dx}{dt}$$

$$u_y = dy/dt$$

$$u_z = dz/dt$$

} in S-frame.

Ily, in S' frame,

$$u'_x = dx'/dt'$$

$$u'_y = dy'/dt'$$

$$u'_z = dz'/dt'$$

} (i)

By diff. the Inverse Lorentz transformation equ for x, y & z & t, we get

$$dx = \frac{dx' + v\,dt'}{\sqrt{1 - v^2/c^2}}$$

$$dy = dy'$$

$$dz = dz'$$

$$dt = \frac{dt' + v\frac{dx'}{c^2}}{\sqrt{1 - v^2/c^2}}$$

} (ii)

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From eqⁿ (i) & (ii),

$$u_x = \frac{dx}{dt} = \frac{dx' + v\,dt'}{dt' + \frac{v\,dx'}{c^2}} \Rightarrow \boxed{u_x = \frac{u'x + v}{1 + \frac{v}{c^2} \cdot u'x}}$$

$$\Rightarrow u_x = \frac{dx'/dt' + v\,dt'/dt'}{1 + \frac{v}{c^2}\frac{dx'}{dt'}}$$

$$\boxed{u_y = \frac{u'y\sqrt{1 - v^2/c^2}}{1 + \frac{v}{c^2} \cdot u'x}}$$

$$\boxed{u_z = \frac{u'z\sqrt{1 - v^2/c^2}}{1 + \frac{v}{c^2} \cdot u'x}}$$

Let us put $\boxed{u'_x = c}$, then,

$$u_x = \frac{c + v}{1 + \frac{v}{c^2} \cdot c} \Rightarrow u_x = \frac{c + v}{\left(\frac{c + v}{c}\right)}$$

$$\Rightarrow \boxed{u_x = c}$$

If an object moves with velocity 'c' w.r.t. other than their relative velocity is always 'c' whatever it may be the velocity of the other.

Again let us put $u'_x = c = v$, then.

$$u_x = \frac{c + c}{1 + \frac{c^2}{c^2}} = \frac{2c}{2}$$

$$\Rightarrow \boxed{u_x = c}$$

This shows that, the addition of velocity of light to the velocity of light only reproduces the velocity of light. It concludes that the velocity of light is an absolute constant.

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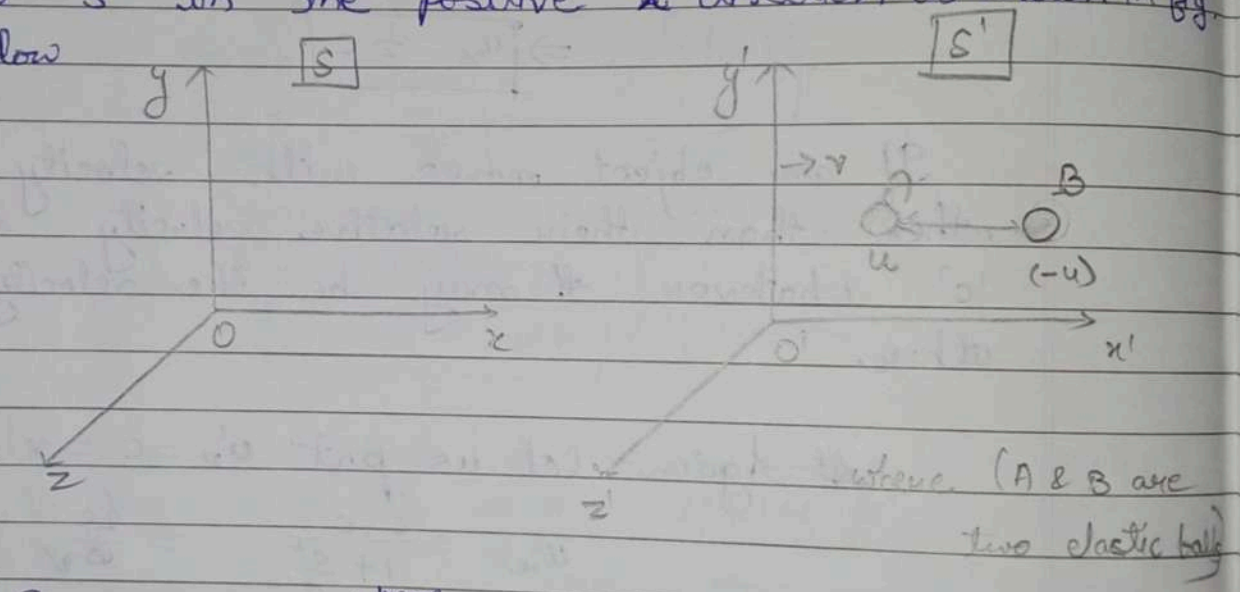
# To prove $\boxed{m = \frac{m_0}{\sqrt{1 - v^2/c^2}}}$

OR

# Variation of mass with velocity

**Relativistic formula for :-**

Let us consider 2 frame of reference S & S' moving with constant velocity v relative to S in the positive x direction as shown in fig. below




where (A & B are two elastic balls)

Suppose in S' frame, two exactly similar elastic balls A & B, each of mass m approach each other with equal speeds (u) & (-u). They collide with each other & come to form a larger group.

Let $u_1$ & $u_2$ are the velocities of balls relative to S then, a/c Lorentz velocity transformation

$$u_1 = \frac{u+v}{1+\frac{uv}{c^2}} \quad \text{---} \quad (1)$$

$$u_2 = \frac{-u+v}{1-\frac{uv}{c^2}} \quad \text{---} \quad (2)$$

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Now, considering the conservation of momentum, we get

$$m_1u_1 + m_2u_2 = (m_1+m_2)v \quad \text{— (3)}$$

$$\Rightarrow m_1\left[\frac{u+v}{1+\frac{uv}{c^2}}\right] + m_2\left[\frac{-u+v}{1-\frac{uv}{c^2}}\right] = (m_1+m_2)v.$$

$$\Rightarrow \frac{m_1u + m_1v}{1+\frac{uv}{c^2}} - \frac{m_2u + m_2v}{1-\frac{uv}{c^2}} = (m_1+m_2)v.$$

$$\Rightarrow \frac{m_1u - \frac{m_1u^2v}{c^2} + m_1v - \frac{m_1v^2}{c^2} - m_2u + \frac{m_2u^2v}{c^2} + m_2v - \frac{m_2v^2}{c^2}}{1^2 - \left(\frac{uv}{c^2}\right)^2} = (m_1+m_2)v.$$

$$\Rightarrow u(m_1-m_2) + v(m_1+m_2) - \frac{u^2v}{c^2}(m_1-m_2) - \frac{v^2}{c^2}(m_1+m_2) = (m_1+m_2)v \times \left[1-\left(\frac{uv}{c^2}\right)^2\right]$$

$$\Rightarrow (m_1-m_2)\left(u - \frac{u^2v}{c^2}\right) + (m_1+m_2)\left(v - \frac{v^2}{c^2}\right) = (m_1+m_2)v \times \left[1-\left(\frac{uv}{c^2}\right)^2\right]$$

not correct way

$$\Rightarrow m_1\left[\frac{u+v}{1+\frac{uv}{c^2}} - v\right] = m_2\left[v - \frac{-u+v}{1-\frac{uv}{c^2}}\right]$$

$$\Rightarrow m_1\left[\frac{u+v-v-\frac{uv^2}{c^2}}{1+\frac{uv}{c^2}}\right] = m_2\left[\frac{v-\frac{uv}{c^2}+u-v}{1-\frac{uv}{c^2}}\right]$$

$$\Rightarrow m_1\left[\frac{u\left(1-\frac{v^2}{c^2}\right)}{1+\frac{uv}{c^2}}\right] = m_2\left[\frac{u\left(1-\frac{v^2}{c^2}\right)}{1-\frac{uv}{c^2}}\right]$$

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$$\boxed{\frac{m_1}{m_2} = \frac{1+\frac{ur}{c^2}}{1-\frac{ur}{c^2}}} \quad \text{---} (4)$$

Also,

$$\Rightarrow 1 - \frac{u_p^2}{c^2} = \frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1+\frac{ur}{c^2}\right)^2} \quad \text{---} (5)$$

$$\Rightarrow 1 - \frac{u_2^2}{c^2} = \frac{\left(1-u^2/c^2\right)\left(1-v^2/c^2\right)}{\left(1-\frac{ur}{c^2}\right)^2} \quad \text{---} (6)$$

Dividing equ (6) by (5), we get

$$\frac{\left(1-u_2^2/c^2\right)}{\left(1-u_1^2/c^2\right)} = \frac{\left(1+ur/c^2\right)^2}{\left(1-\frac{ur}{c^2}\right)^2}$$

$$\Rightarrow \frac{1+\frac{ur}{c^2}}{1-ur/c^2} = \frac{\sqrt{1-u_2^2/c^2}}{\sqrt{1-u_1^2/c^2}} \quad \text{---} (7)$$

From equ (4) & (7), we get

$$\frac{m_1}{m_2} = \frac{\sqrt{1-u_2^2/c^2}}{\sqrt{1-u_1^2/c^2}}$$

$$\Rightarrow m_1\left(\sqrt{1-u_1^2/c^2}\right) = m_2\sqrt{1-u_2^2/c^2} \quad \text{---} (8)$$

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From equ (8) it is clear that the LHS & RHS are independent of one another & this results may be true only if each is a constant.

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∴ We can write, $m_1\sqrt{1-v_1^2/c^2} = m_2\sqrt{1-v_2^2/c^2} = m_0$

Where, $m_0$ = Rest mass of the body corresponds to zero velocity, Thus, $\boxed{m_1 = \frac{m_0}{\sqrt{1-v_1^2/c^2}}}$

If $m$ be the mass of the body when it is moving with velocity 'v' then in general we can write,

$$\boxed{m = \frac{m_0}{\sqrt{1-v^2/c^2}}} \quad \text{——— (9)}$$

it is relativistic formula for the variation of mass with velocity.

VVI

# Mass - Energy Equivalence :-

To prove $\boxed{E = mc^2}$

⇒ "Energy" may appear as "mass" & "mass" may appear as "Energy".

Let us consider a particle of mass 'm' acted upon by a force, 'F' in the same direction as its velocity 'v' & the force displaces the body through a distance 'dx'. Then the increase in the K.E. ($dK$) $\boxed{(dE_K) = \text{work done}}$

The force is defined as the rate of change of momentum.

So, $$F = \frac{d(mv)}{dt} = \frac{m\,dv}{dt} + v\frac{dm}{dt} \quad \text{——— (i)}$$

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But, according to the theory of relativity, both mass & velocity are variable.

$$dE_k = m\frac{dv}{dt}dx + v\frac{dm}{dt}dx \quad \left[\because \frac{dx}{dt} = v\right]$$

$$\Rightarrow \boxed{dE_k = mv\,dv + v^2 dm} \quad \text{——} \quad (2).$$

A/c to variation of mass with velocity,

$$\boxed{m = m_0/\sqrt{1 - v^2/c^2}} \quad \text{——} \quad (3)$$

On squaring these equo (3), we get

$$\boxed{m^2 = \frac{m_0^2}{1 - v^2/c^2} = \frac{m_0^2 c^2}{c^2 - v^2}}$$

$$\Rightarrow m^2c^2 - m^2v^2 = m_0^2c^2.$$

$$\Rightarrow \boxed{m^2c^2 = m_0^2c^2 + m^2v^2}.$$

Diff the above equo, we get

$$c^2 \cdot 2m\,dm = 0 + m^2 2v\,dv + v^2 2m\,dm.$$

$$\Rightarrow 2c^2 dm = 2mv\,dv + 2v^2 dm$$

$$\Rightarrow c^2 \cdot dm = mv\,dv + v^2 dm. \quad \text{——} \quad (4)$$

From equo (2) & (4),

$$dE_k = c^2 dm \quad \text{——} \quad (5)$$

Thus, the change in Kinetic energy $dE_k$ is directly proportional to a change in mass $dm$.

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Integrating equ ⑤ with taking the proper limits,

$$\int_0^{E_k} dE_k = c^2 \int_{m_0}^{m} dm$$

$$\Rightarrow E_k = c^2 (m - m_0).$$

$$\Rightarrow \boxed{E_k = mc^2 - m_0c^2} \longleftarrow (6)$$

This is relativistic formula for K.E. when the body is at rest. Energy stored in the body is $m_0c^2$, which is called the rest mass energy.

Thus the total energy (E) is the sum of K.E & rest mass energy.

$$\therefore E = E_k + m_0c^2$$

$$\Rightarrow E = mc^2 - m_0c^2 + m_0c^2.$$

$$\Rightarrow \boxed{E = mc^2}$$ proved

This is Einstein mass-energy relation. This relation shows mass may appear as energy and energy may appear as mass

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# Ch-2 Electromagnetic Theory

## (Emf Theory)

maxwell field eqns :- 1) Differential form 2) Integral form 3) Free space

* del operator * gradient * divergent * Curl
* Equ of Continuity * Stoke's Theorem * Divergent Theorem

1) $\nabla \cdot D = \rho$ ⇒ Gauss Law in Electrostatics
2) $\nabla \cdot B = 0$ ⇒ Gauss law for magnetism
3) $\nabla \times E = -\frac{\partial \vec{B}}{\partial t}$ ⇒ Faraday's Law in diff. form
4) $\nabla \times H = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ ⇒ Ampere's Circuital law

ampere's circuital law → ampere's modified circuital law

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* Del operator :- The del operator is the differential operators, which is represented by $\vec{\nabla}$ and is represented by $\boxed{\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}}$.

* Gradient :- Gradient is the rate of change of quantity with distance. For a function $F(x,y,z)$ of three variables, we obtain from a theorem on partial derivative

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial z}dz$$

gradient of $\phi$, $\nabla\phi$ $= \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$

or

$\Rightarrow dF = \vec{\nabla}F \cdot d\vec{r}$ where,

$\Rightarrow \boxed{\vec{\nabla}F = \hat{i}\frac{\partial F}{\partial x} + \hat{j}\frac{\partial F}{\partial y} + \hat{k}\frac{\partial F}{\partial z} = \text{gradient of } F}$

It is a vector.

* Divergence :- The divergence of $\vec{A}$ is defined as the net outward flux per unit volume over a closed surface. Actually, the divergence of $\vec{A}$ at a given point is a

measure of how much the vector $\vec{A}$ spreads out i.e., diverges, from that point. It is denoted by $\vec{\nabla}\cdot\vec{A}$. It is scalar.

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

* **Curl :-** The curl of vector field $\vec{A}$ is represented by $\vec{\nabla}\times\vec{A}$. So, it is given by

$$\vec{\nabla}\times\vec{A} = i\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{j}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

* **Equation of continuity :-** The equation of continuity says that the total current flowing out of some volume must be equal to the rate of decrease of the charge within that volume, if the charge is neither being created nor lost.

H.W.

$$\text{div}.\vec{J} + \frac{\partial \rho}{\partial t} = 0$$

* **Stoke's Theorem :-**

H.W.

$$\int_l \vec{A}\cdot d\vec{l} = \int_s \text{Curl}\,\vec{A}\cdot d\vec{s}$$

or

$$\int_l \vec{A}\cdot d\vec{l} = \int (\vec{\nabla}\times\vec{A})\, ds$$

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# Gauss Divergence Theorem :-

$$\int_S \vec{A} \cdot ds = \int_V \text{div}\, \vec{A} \cdot dv \quad \text{or} \quad \int_V \nabla \vec{A} \, dv$$

# Vector Identities

1) → $\nabla \cdot \nabla \times H = 0$ ( div. of $\nabla \times H = 0$ )

2) $\nabla \times \nabla \times H = \vec{\nabla}(\vec{\nabla} \cdot \vec{H}) - \vec{\nabla}^2 H$ [Curl of curl ×H

3)

## Material Science of Engineering

## Dielectric Polarization.

H.W. 1) $B = \mu_0 H + M$

2) $D = \varepsilon_0 E + P$

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# Maxwell's Field Equations

Maxwell

1) differential form —

(i) $\nabla . D = \rho$.

(ii) $\nabla . B = 0$.

(iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$.

(iv) $\nabla \times H = \vec{J} + \frac{\partial \vec{D}}{\partial t}$.

# Reduction of Maxwell's 1st Equ

According to Gaussses law

The total outward normal electric flux over a closed surface is equal to $(1/\varepsilon_0)$ times the total charge enclosed by the surface.

Mathematically, we can write,

$$\oint_S \vec{E} . ds = \frac{q}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int_V \rho \, dv \quad \text{— (A)}$$

$\Rightarrow$ where, $q = \int_V \rho \, dv$ = volume charge Integral

Now, we have to use Gaussses divergence theorem,

$$\oint_S \vec{E} . ds = \frac{1}{\varepsilon_0} \int_V \rho \, dv \quad \text{— (A)}$$

Proof :-

$\Rightarrow$ L.H.S $= \oint_S \vec{E} . ds = \int_V . div . E \, dv = \int_V \nabla . E \, dv$ — (B)

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From equ (A) & (B),

$$\int_V \text{div}.E\, dv = \frac{1}{\varepsilon_0}\int_V \rho\, dv$$

$$\Rightarrow \int_V \text{div}.\vec{D}\, dv = \int_V \rho\, dv$$

$$\Rightarrow \int_V (\text{div}\,\vec{D} - \rho)\, dv = 0$$

$$\Rightarrow \int_V \boxed{\text{div}\,\vec{D} - \rho = 0}$$

$$\Rightarrow \text{div}\,\vec{D} = \rho$$

$$\Rightarrow \boxed{\nabla.D = \rho}$$

# Reduction of Maxwell's 2nd Equation.

$$\boxed{\nabla.\vec{B} = 0}$$

This is Gaussses law for magnetic field in differential form.

B = magnetic Induction field.

$\phi$ = magnetic flux

$$\boxed{\phi = \oint_S \vec{B}.ds} \quad \text{---} \quad (a)$$

$$\boxed{\oint_S \vec{B}.ds = 0} \quad \text{---} \quad (b)$$

Applying Gaussses divergence theorem to transform surface integral into volume integral

$$\boxed{\int_V \text{div}\,\vec{B}.\, dv = 0}$$

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$$\boxed{\text{div}.\vec{B} = 0 \quad \text{or} \quad \nabla.B = 0}$$ ☆

# Curl Maxwell's 3rd differential equ :-

This is actually Faraday's law in diff. equ.

According to Faraday's law of EMI, whenever, there is a change in the magnetic flux link with a coil, there is an Emf induced in the coil. This induced emf is directly proportional to rate of change of magnetic flux.

So, we can write,

$$\text{Ind. Emf} \propto -\frac{d\phi}{dt}.$$

OR

$$e = -\frac{d\phi}{dt} \quad \text{— (i)}$$

But also, we can write,

$$e = \oint_l \vec{E}.dl \longrightarrow \text{(ii)}$$

where, e = Electric intensity

From equ (i) & (ii),

$$\oint_l \vec{E}.dl = -\frac{d\phi}{dt} \quad \text{— (3)}$$

Also, we know that,

magnetic flux, $\phi = \oint_S \vec{B}.ds$ — (4)

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From equo (3) & (4),

$$\oint_l \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\oint \vec{B}\cdot d\vec{s}$$

$$\Rightarrow \oint_l \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} ds \quad \text{—— (5)}$$

Transforming the line integral into surface integral, we get by using Stokers theorem, we get

$$\oint_l \vec{E}\cdot d\vec{l} = \oint_S (\nabla \times E) ds \quad \text{—— (6)}$$

From equo (5) & (6),

$$\oint_S (\nabla \times E) ds = -\oint_S \frac{\partial \vec{B}}{\partial t} ds$$

$$\Rightarrow \int \left(\nabla \times E + \frac{\partial \vec{B}}{\partial t}\right) ds = 0$$

$$\Rightarrow (\nabla \times E) + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\Rightarrow \boxed{\nabla \times E = -\frac{\partial \vec{B}}{\partial t}} \quad \text{—— (7)}$$

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Emf Theory :-

Maxwell field equ· (differential form).

# Maxwell's 4th differential field equation :-

This is actually ampere's law in modified form.
According to Amperes circuital law,

$$\oint_l \vec{B} \cdot dl = \mu_0 I \longrightarrow (1)$$

Equ· (1) explains that the work done in taking the unit magnetic pole once around the closed loop is $\mu_0$ times the currents linked with the surface.

$$\therefore \quad I = \int_S \vec{J} \cdot ds \quad — (2)$$

From equ· (1) & (2)

$$\oint \vec{B} \cdot dl = \mu_0 \oint_S \vec{J}\, ds$$

$$\Rightarrow \oint \frac{B}{\mu_0} dl = \oint_S J\, ds.$$

$$\Rightarrow \oint_l H\, dl = \oint_S J\, ds. \quad [\because B = \mu_0 H + M] \quad — (3)$$

Transforming the line integral into surface integral by using Stoke's theorem, we get

$$\oint_l H \cdot dl = \oint_S (\nabla \times H)\, ds. \quad — (4)$$

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From equo (3) & (4),

$$\boxed{(\vec{\nabla} \times \vec{H}) = \vec{J}} \quad \text{---(5)}$$

it is not applicable for time-varying field

According to Ampere's law

Let us examine this equo, for the time varying case.

Taking divergence of equo (5) both sides, we get

$$\boxed{\vec{\nabla} \cdot \nabla \times H = \vec{\nabla} \cdot \vec{J}}$$

$$\Rightarrow \nabla \cdot \nabla \times H = 0 \quad \Rightarrow \nabla \cdot J = 0 \quad \text{---(6)}$$

But, also we know that

$$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$$

$$\Rightarrow 0 + \frac{\partial \rho}{\partial t} = 0 \quad \text{[by continuity]}$$

$$\Rightarrow \boxed{\frac{\partial \rho}{\partial t} = 0}$$

Further, $\rho$ is a constant

So, field is static.

# Deduction of modified ampere's circuital law :-

$$\boxed{\nabla \times H = J + J_d} \quad \text{---(8)}$$

Proof :- Taking divergence both sides in equo (8) we get

$$\vec{\nabla} \cdot \nabla \times H = \vec{\nabla} \cdot (J + J_d)$$

Further $\nabla \cdot J_d = -\vec{\nabla} \cdot J$ [$\because \nabla \cdot \nabla \times H = 0$]

Again from equo of continuity :-

$$\boxed{\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}}$$

$\Rightarrow J = \frac{\delta \rho}{\delta t} \Rightarrow \rho = 0$

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$\nabla \cdot J_d = \nabla \frac{\partial D}{\partial t}$

$\Rightarrow \nabla \cdot J_d = \frac{\partial}{\partial t}(\nabla \cdot D) \Rightarrow \boxed{J_d = \frac{\partial D}{\partial t}}$ — (7)

So, from equ (6) & (7), $\boxed{\nabla \times H = J + \frac{\partial D}{\partial t}}$ proved

(This is maxwell 4th equ.)

## Maxwell field equation in Integral Form

1) $\oint_S \vec{D} \cdot dl = \oint \rho dv$

2) $\oint_S \vec{B} \cdot ds = 0.$

3) $\oint_l B \cdot dl = -\oint_S \frac{\partial \vec{B}}{\partial t} ds.$

4) $\oint_l \vec{H} \cdot dl = \oint_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) ds.$

① Deduction of Maxwell's 1st Integral Equation

② Proof:- $\because \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ or $\vec{\nabla} \cdot \vec{D} = \rho$.

On integrating both sides over any volume V, we get

$$\oint_V \vec{\nabla} \cdot D \, dv = \oint \rho \, dv \quad \text{— (i)}$$

Using Gauss's Divergence theorem

$$\oint_V \text{div} \vec{D} \cdot dv = \oint_S D \cdot ds \quad \text{— (2)}$$

From equ (1) & (2),

$$\boxed{\oint_S \vec{D} \cdot ds = \int \rho \, dv}$$ proved

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(2.) Maxwell's 2nd Integral Equation

We know that Maxwell's 2nd differential form

$$\vec{\nabla} \cdot \vec{B} = 0.$$

Applying volume integral both sides,

$$\oint_V (\vec{\nabla} \cdot \vec{B}) dv = 0 \quad \text{—— (A)}$$

Apply Gauss Divergence Theorem, we get

$$\oint_V (\vec{\nabla} \cdot \vec{B}) dv = \oint_S \vec{B} \cdot \vec{ds} \quad \text{—— (B)}$$

From equns (A) & (B), we get

$$\oint_S (\vec{B} \cdot \vec{ds}) = 0$$

proved

(3.) Maxwell's 3rd Integral Equation

We know that Maxwell's 3rd differential equn

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}.$$

Apply surface integral both sides

$$\oint_S (\vec{\nabla} \times \vec{E}) ds = -\oint_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds} \quad \text{—— (A)}$$

Apply Stoke's theorem, we get

$$\oint_S (\vec{\nabla} \times \vec{E}) ds = \oint_L \vec{E} \cdot dl \quad \text{—— (B)}$$

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From equo (A) & (B)

$$\oint_l \vec{E} \cdot d\vec{l} = -\oint_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

proved

(4.) Maxwell's 4th Integral Equation :-

We know that Maxwell's 4th differential from equo.

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Apply surface integral on both sides

$$\oint_s (\vec{\nabla} \times \vec{H}) \cdot d\vec{s} = \oint_s \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \cdot d\vec{s} \quad \text{—— (A)}$$

Apply stooke's theorem

$$\oint_s (\vec{\nabla} \times \vec{H}) \cdot d\vec{s} = \oint_l \vec{H} \cdot d\vec{l} \quad \text{—— (B)}$$

From equo (A) & (B)

$$\oint_l \vec{H} \cdot d\vec{l} = \oint_s \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \cdot d\vec{s}$$

proved

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# Propagation of Electromagnetic field in vacuum or free space.

For free space, VVI

$\nabla \cdot D = \rho$ — (1)
$\nabla \cdot B = 0$ — (2)
$\nabla \times E = -\frac{\partial B}{\partial t}$ — (3)
$\nabla \times H = J + \frac{\partial D}{\partial t}$ — (4)

$\rho = 0$, $J = 0$ & $\varepsilon_r = 1$, $\mu_r = 1$ | $D = \varepsilon_0 E$, $B = \mu_0 H$ | $\varepsilon_r = 1$, $\mu_r = 1$

$\Rightarrow$ $\nabla \cdot E = 0$ — (5)
$\nabla \cdot H = 0$ — (6)
$\nabla \times E = -\mu_0 \frac{\partial H}{\partial t}$ — (7)

$\nabla \times H = \varepsilon_0 \frac{\partial E}{\partial t}$ — (8)

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## Expression for (c) :-

Taking curl on both sides of equ (7), we get

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H})$$

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial}{\partial t} \left(\varepsilon_0 \frac{\partial E}{\partial t}\right)} \quad [\text{using equ (8)}]$$

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}} \quad — (9)$$

Also, $\boxed{\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \nabla(\vec{\nabla} \cdot \vec{E}) = \vec{\nabla}^2 E}$

Using equ (5) $\Rightarrow \boxed{\vec{\nabla} \times \vec{\nabla} \times \vec{E} = 0 - \vec{\nabla}^2 E}$ — (10)

From equ (9) & (10), we get

$$\boxed{\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}} \quad — (11)$$

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From equo (12) & (13), we get

$$\Rightarrow \boxed{\nabla^2 H = \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2}} \quad \Leftarrow \nabla \times \nabla \times H = -\mu_0 \varepsilon_0 \frac{\delta^2 H}{\delta t^2} \quad -(12)$$

$$\Leftarrow \nabla \times \nabla \times H = -\vec{\nabla}^2 H \quad \rightarrow (13)$$

Equo (11) & (14) are the wave equos for electric & magnetic field respectively.

# Propagation of Electromagnetic wave :-

Let us consider equo of wave (general equo of wave)

$$y = a \sin \frac{2\pi}{\lambda}(ct - x) \quad — (1)$$

$$y = f(x, t)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2\pi}{\lambda} a \cos \frac{2\pi}{\lambda}(ct - x)$$

$$\frac{d^2 y}{dx^2} = \left(-\frac{2\pi}{\lambda}\right)^2 (-a) \sin \frac{2\pi}{\lambda}(ct - x).$$

$$\Rightarrow \frac{\partial^2 y}{\partial x^2} = -\frac{4\pi^2}{\lambda^2} y \quad — (2)$$

Wy,

$$\frac{\partial^2 y}{\partial t^2} = -\frac{4\pi^2}{\lambda^2} c^2 y \quad — (3).$$

$$\therefore \frac{\partial^2 y}{\partial x^2} \Big/ \frac{\partial^2 y}{\partial t^2} = \frac{1}{c^2} \Rightarrow \boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2}}$$

$$\Rightarrow \boxed{\vec{\nabla}^2 y = \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2}} \quad — (4)$$

After comparing equos (11), (12) & above equo (4), we get

$$\Rightarrow \frac{1}{c^2} = \mu_0 \varepsilon_0 \quad \Rightarrow c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$\Rightarrow c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$$

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$\Rightarrow \boxed{\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}}$

$\boxed{\varepsilon_0 = 8.85 \times 10^{-12}\ Em^{-1}}$

After putting these two values in $c^2 = \frac{1}{\mu_0 \varepsilon_0}$ we get

$$\boxed{c = 3 \times 10^8\ m/s}$$

# Poynting Theorem & Poynting Vector

or

## Electromagnetic Energy

The propagation of electromagnetic wave results in the transportation of energy from one place to other.

Now according to Maxwell 3rd & 4th differential equ

$$\text{curl } E = -\frac{\partial B}{\partial t}$$

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \quad \text{—— (1)} \quad \& \quad \boxed{\vec{\nabla} \times H = \vec{J} + \frac{\partial \vec{D}}{\partial t}} \quad \text{—— (2)}$$

Taking scalar product of equ (1) with H & equ (ii) with E, we get

$$\boxed{H \cdot \vec{\nabla} \times \vec{E} = -H \frac{\partial \vec{B}}{\partial t}} \quad \text{—— (3)}$$

$$\& \quad \boxed{E \cdot \vec{\nabla} \times \vec{H} = \vec{E}\vec{J} + \vec{E}\frac{\partial \vec{D}}{\partial t}} \quad \text{—— (4)}$$

Subtracting equ (4) from equ (3), we get

$$H \cdot \vec{\nabla} \times \vec{E} - E \cdot \vec{\nabla} \times \vec{H} = -H \frac{\partial \vec{B}}{\partial t} - E \frac{\partial \vec{D}}{\partial t} - \vec{E}\vec{J}$$

$$\Rightarrow \boxed{H \cdot \vec{\nabla} \times \vec{E} - E \cdot \vec{\nabla} \times \vec{H} = -\left(H \frac{\partial \vec{B}}{\partial t} + E \frac{\partial \vec{D}}{\partial t}\right) - \vec{E}\vec{J}} \quad \text{—— (5)}$$

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Now using vector identity —

$$\boxed{\text{div.}(E \times H) = H(\nabla \times E) - E(\nabla \times H)} \quad \text{—— (5-a)}$$

From equo (5) & (5-a), we get

$$\boxed{\text{div.}(E \times H) = -\left(H \cdot \frac{\partial \vec{B}}{\partial t} + E \frac{\partial \vec{D}}{\partial t}\right) - EJ} \quad \text{— (6)}$$

$\because$ $\boxed{B = \mu_0 H = \mu H}$

& $\boxed{D = \varepsilon_0 E = \varepsilon E}$

$$\therefore \boxed{\text{div.}(E \times H) = -\left(H \cdot \frac{\partial(\mu H)}{\partial t} + E \cdot \frac{\partial(\varepsilon E)}{\partial t}\right) - EJ}$$

$$\Rightarrow \text{div}(E \times H) = -\left\{\frac{1}{2}\frac{\mu \partial(H^2)}{\partial t} + \frac{1}{2}\varepsilon \frac{\partial(E^2)}{\partial t}\right\} - EJ$$

$$\Rightarrow \text{div}(E \times H) = -\left\{\frac{1}{2}\frac{\partial}{\partial t}(H \mu H) + \frac{1}{2}\frac{\partial}{\partial t}(E \cdot D)\right\} - EJ.$$

$$\Rightarrow \text{div}(E \times H) = -\frac{\partial}{\partial t}\left[\frac{1}{2}(E \cdot D + H \cdot B)\right] - EJ \quad \text{—— (7)}$$

Integrating equo (7) over a volume 'v' bounded by a surface 's', we get —

$$\int_v \text{div}(E \times H) = -\int_v \left\{\frac{\partial}{\partial t}\frac{1}{2}(E \cdot D + H \cdot B)\right\} dv - \int_v E \cdot J \, dv$$

Applying Gauss's divergence theorem in above equo

$$\int_s (E \times H) \, ds = -\frac{\partial}{\partial t}\int_v \frac{1}{2}(E \cdot D + H \cdot B) \, dv - \int_v J \cdot E \, dv$$

$$\Rightarrow \boxed{-\int_v J \cdot E \, dv = \frac{\partial}{\partial t}\int_v \frac{1}{2}(E \cdot D + H \cdot B) \, dv + \int_s (E \times H) \, ds} \quad \text{— (8)}$$

(1) term (2) term (3) term

5 marks unit test
5 marks project
20 marks mid term.

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**The first term :→**

It represents rate of transfer of energy into the electromagnetic field due to the motion of charge.

The 2nd term represents the rate of electromagnetic energy stored.

The 3rd term represents the amount of energy crossing per second through the closed surface.

$$\Rightarrow -E \cdot \boxed{(E \times H) = S} \Rightarrow$$ It is called Poynting Theorem.

Equ (8) represents the law of conservation of electromagnetic energy.

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4/12/23 Ch-3

# Quantum Physics.

* Wave function ($\psi$)
* Physical meaning of wave function.

$|\psi|^2$ = Normalization Condition

* Light wave (Maxwell Field Equation)
* Matter wave (Schrodinger wave equation)
  - Time Independent Wave eqn
  - Time dependent wave eqn

$$|\psi|^2 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dx\,dy\,dz = 1 \Rightarrow\Rightarrow \text{(Probability density)}$$

$$|\psi|^2 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dx\,dy\,dz = 0$$

$$\int_{-\infty}^{+\infty} dx = 1 \qquad \int_{-\infty}^{+\infty} dx = 0$$

## # Wave Function ($\psi$) :-

The wave function ($\psi$) associated with the matter waves. has no direct physical interpretation but if we find the probability density ($|\psi|^2$) then its has physical meaning.

## ⇒ Physical meaning of a wave function :-

(i) In the matter wave, the quantity that varies periodically, is called the wave function ($\psi$).

(ii) The wave function ($\psi$) is a complex quantity that means it includes both real & imaginary parts. Hence, $\psi$ by itself can't be an observable quantity.

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(iii) The probability density $|\psi|^2 = \psi^* \psi$ is a real & +ve quantity and is measurable where $\psi^*$ is the complex conjugate of $\psi$.

(iv) The probability density $|\psi|^2$ is the probability of finding the particle in a unit volume and it can have values anywhere b/w '0' & '1'.

So,

Normalization condition

$$\iiint \psi^* \psi \, dv = 1 \Rightarrow \text{Particle is present}$$

→ (Probability to penetrate the barrier by electrons)

$$\iiint \psi^* \psi \, dv = 0 \Rightarrow \text{Particle is absent.}$$

# Characteristic of Wave Function

(i) '$\psi$' must be finite, continuous & single valued.

(ii) $\psi$ must be normalizable.

Time.

⇒ Schrodinger's Independent Wave Equation :-

Schrodinger derive an equation for the motion of matter wave, he made the following assumptions :—

(i) He assumed that the de Broglie wavelength holds good for any particle moving any field of force with potential energy 'V', then the sum of kinetic energy & potential energy is given by $E = \frac{1}{2}mv^2 + V$

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$$E = \frac{p^2}{2m} + V$$

$$(E-V) = \frac{p^2}{2m} \Rightarrow p^2 = 2m(E-V) \Rightarrow p = [2m(E-V)]^{1/2}$$

De Broglie wavelength :-

$$\lambda = \frac{h}{p} = \frac{h}{[2m(E-V)]^{1/2}} \quad \text{— (i)}$$

(ii) Ide assumed that the wave fn $\psi$ is governed by the following equo :-

$$\psi = \psi_0 e^{-i\omega t} \quad \text{— (ii)}$$

wave

The fn $\psi$ is also called probability amplitude.

The usual wave equo in Cartesian co-ordinate form is given by

General equation of Cartesian wave equo =

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{— (iii)}$$

where,

$\omega$ = angular frequency

$u$ = wave velocity

also, $\omega = 2\pi\nu$

$u = \nu\lambda$

Diff equo (ii) partially, we get

$$\frac{\partial\psi}{\partial t} = -i\omega\psi_0 e^{-i\omega t} = -i\omega t\psi_0$$

$$\therefore \frac{\partial^2\psi}{\partial t^2} = i^2\omega^2 t^2 \psi_0 e^{-i\omega t}$$

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2\psi_0 e^{-i\omega t} \quad \text{— (iv)} \quad [\because \psi = \psi_0 e^{-i\omega t}]$$

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Putting value of $\frac{\partial^2\psi}{\partial t^2}$ in equ (iii),

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = -\left(\frac{\omega^2}{\mu^2}\right)\psi \Rightarrow \boxed{\frac{\omega^2}{\mu^2} = \left(\frac{2\pi\nu}{\nu\lambda}\right)^2}$$

$$\Rightarrow \boxed{\frac{\omega^2}{\mu^2} = \frac{4\pi^2}{\lambda^2}} \Rightarrow \boxed{\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = -\left(\frac{4\pi^2}{\lambda^2}\right)\psi}$$

$$\Rightarrow \boxed{\nabla^2\psi + \frac{4\pi^2\psi}{\lambda^2} = 0} \quad \text{—— (iv)}$$

$$\Rightarrow \boxed{\lambda = \frac{h}{p} = \frac{h}{[2m(E-V)]^{1/2}}}$$

Putting the value of $\lambda$ in equ (iv), we get

$$\nabla^2\psi + 4\pi^2 \frac{[2m(E-V)]\psi}{h^2} = 0.$$

$$\Rightarrow \nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0 \quad \text{—— (v)}$$

$$\Rightarrow \nabla^2\psi + 2\left(\frac{2\pi}{h}\right)^2 m(E-V)\psi = 0$$

But, $\hbar = \frac{h}{2\pi}$

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\cdot\psi = 0.} \quad \text{—— (vi)}$$

In case of free particles, no external force is acting on it

$$\therefore \boxed{V = 0}$$

Equ (6) is also known as Schrodinger time independent wave equ.

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}E\psi = 0} \quad \text{—— (vii)}$$

This equ is Schrodinger wave equ in free particle.

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# Schrodinger Time dependent wave Equation:-

$$\boxed{\psi = \psi_0 e^{-i\omega t}} \quad \text{--- (ii) (from equ (ii))}$$

$$\Rightarrow \frac{d\psi}{dt} = -i\omega \psi_0 e^{-i\omega t} \Rightarrow \boxed{\frac{d\psi}{dt} = -i 2\pi\nu \psi} \quad [\because \omega = 2\pi\nu]$$

$$\because E = h\nu \Rightarrow \boxed{\nu = \frac{E}{h}}$$

$$\Rightarrow \therefore \frac{d\psi}{dt} = -i\left(2\pi \frac{E}{h}\right)\psi.$$

$$\Rightarrow E\psi = \frac{1}{-i}\left(\frac{h}{2\pi}\right)\frac{d\psi}{dt}.$$

$$\Rightarrow \boxed{E\psi = \frac{-2\hbar}{-i}\frac{d\psi}{dt}} = \frac{(i^2)2\hbar}{i}\frac{d\psi}{dt} \quad \text{--- (3)}$$

According to Schrodinger time independent wave equ,

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0}$$

$$\nabla^2\psi + \frac{2m}{\hbar^2} E\psi - \frac{2m}{\hbar^2} V\psi = 0 \quad \text{--- (4)}$$

$$\nabla^2\psi + \frac{2m}{\hbar^2} i\hbar\frac{\partial\psi}{\partial t} - \frac{2m}{\hbar} V\psi = 0.$$

$$\nabla^2\psi - \frac{2m}{\hbar} V\psi = \left(\frac{-2m}{\hbar^2}\right) i\hbar \frac{\partial\psi}{\partial t}$$

$$\Rightarrow \nabla^2\psi - \frac{2m}{\hbar^2} V\psi = \left(\frac{-2m}{\hbar^2}\right) i\hbar \frac{\partial\psi}{\partial t}$$

Multiplying & dividing $(-\hbar^2)$ by $2m$ we get

$$\boxed{\frac{-\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}} \quad \text{--- (5)} \rightarrow \boxed{\text{Time dependent Equ}}$$

Equ (5) can also be written as

$$\left[\frac{\hbar^2}{2m}\nabla^2 + V\right] = H = \text{(Hamiltonian operator)}$$

$$i\hbar\frac{\partial}{\partial t} = E = \text{(Energy operator)}$$

$$E\psi = H\psi \quad \text{——} (6)$$

08/12/23 II

## Application of Schrodinger Equations :—

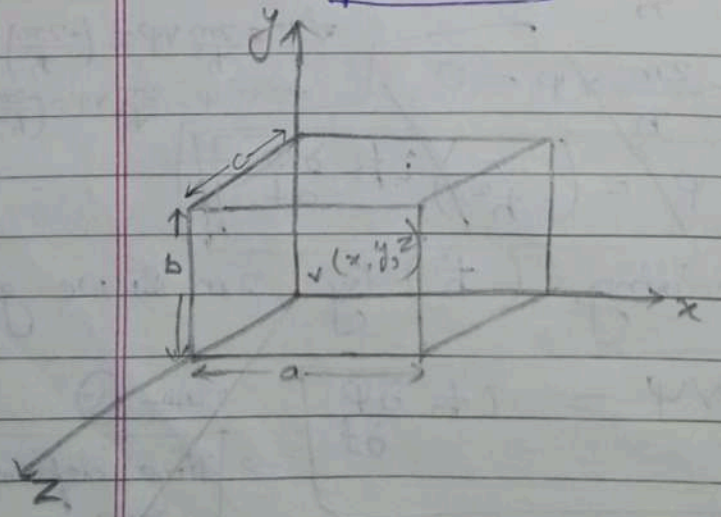
i) Particle in 3-D box
ii) Partical in 1-D box.

→ Eigen Values & Eigen function
(1) Independent (2) Dependent

II Particle in 3-D box :—
No. of probable electrons $(x, t)$

→ $\nabla^2 + \frac{2m}{\hbar^2}(E-V)\psi = 0$ —— (7)

$E\psi = H\psi$ Single Valued Continued Functio

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# Particle in 3-D box

Let us consider a rectangle box having sides a, b & c.

The potential function $V(x,y,z)$ has constant value 0. in the following region.

$$V(x,y,z) = 0 \quad ; \quad 0 \le x \le a$$
$$V(x,y,z) = 0 \quad ; \quad 0 \le y \le b$$
$$V(x,y,z) = 0 \quad ; \quad 0 \le z \le c$$

Potential outside box $V(x,y,z) = \infty$
Potential $V(x,y,z)$ inside box $= 0$.

A/C. Schrodingers Time Independent Wave Equation,

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0} \quad \text{— (1)}$$

Inside the potential, $\boxed{V=0}$, i.e, $\boxed{\nabla V = 0}$

From equ (1),

$$\nabla^2\psi + \frac{2m}{\hbar^2}E\psi = 0$$

$$\boxed{\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{2mE\psi}{\hbar^2} = 0} \quad \text{— (2)}$$

Let us suppose that follow the method of separation of variable,

$$\psi(x,y,z) = X(x)\cdot Y(y)\cdot Z(z)$$

$$\boxed{\psi(x,y,z) = X.Y.Z.} \quad \text{— (3)}$$ (It is the form of double diffraction)

$$\boxed{\frac{\partial^2\psi}{\partial x^2} = YZ\frac{\partial^2 X}{\partial x^2}}, \quad \boxed{\frac{\partial^2\psi}{\partial y^2} = XZ\frac{\partial^2 Y}{\partial y^2}} \quad \& \quad \boxed{\frac{\partial^2\psi}{\partial z^2} = XY\frac{\partial^2 Z}{\partial z^2}} \quad \text{— (4)}$$

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Putting the value of equo (4) & (3) in equo (2),

$$\Rightarrow YZ\frac{\partial^2 X}{\partial x^2} + XZ\frac{\partial^2 Y}{\partial y^2} + XY\frac{\partial^2 Z}{\partial z^2} + \frac{2mE}{\hbar^2}\psi XYZ = 0.$$

Dividing both sides by XYZ, we get

$$\Rightarrow \frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} + \frac{2mE}{\hbar^2} = 0 \quad \text{— (5)}$$

$$\therefore \begin{cases} \frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -\alpha_1^2 \\ \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -\alpha_2^2 \\ \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = -\alpha_3^2 \end{cases} \text{— (6)} \qquad \begin{cases} \frac{\partial^2 X}{\partial x^2} + \alpha_1^2 X = 0 \text{ — (7)} \\ \frac{\partial^2 Y}{\partial y^2} + \alpha_2^2 Y = 0 \text{ — (8)} \\ \frac{\partial^2 Z}{\partial z^2} + \alpha_3^2 Z = 0 \text{ — (9)} \end{cases}$$

$$\boxed{+\alpha_1^2 + \alpha_2^2 + \alpha_3^2 = \frac{2mE}{\hbar^2}} \quad \text{— (10)}$$

General equo of equo (7), (8) & (9) are :—

$$X = A_1\cos\alpha_1 x + B_1\sin\alpha_1 x \quad \text{— (11)}$$
$$Y = A_2\cos\alpha_2 y + B_2\sin\alpha_2 y \quad \text{— (12)}$$
$$Z = A_3\cos\alpha_3 z + B_3\sin\alpha_3 z \quad \text{— (13)}$$

Where $A_1, A_2, A_3$ and $B_1, B_2$ & $B_3$ are constants to be found by applying boundary condition

$$\boxed{\psi(0,y,z) = \psi(x,0,z) = \psi(x,y,0)}$$
$$\boxed{\psi(a,y,z) = \psi(x,b,z) = \psi(x,y,c)}$$

Further if we consider $\boxed{A_1 = A_2 = A_3 = 0}$, then

$$B_1\sin\alpha_1 a = 0$$
$$\Rightarrow \alpha_1 a = n_x\pi, \quad \boxed{\alpha_1 = \frac{n_x\pi}{a}}, \quad \boxed{\alpha_2 = \frac{n_y\pi}{b}}$$

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$$\alpha_3 = \frac{n_z \pi_z}{c}$$

From equ (11), (12) & (13),

$$X = B_1 \sin \frac{n_x \pi x}{a}, \quad Y = \frac{B_2 \sin n_y \pi y}{b}$$

$$Z = \frac{B_3 \sin n_z \pi z}{c} \quad \Big\} \text{ — (14)}$$

$$\psi(x, y, z) = B_1 \cdot B_2 \cdot B_3 \cdot \sin \frac{n_x \pi x}{a} \cdot \frac{\sin n_y \pi y}{b} \cdot \frac{\sin n_z \pi z}{c}$$

$$\psi = k \sin \frac{n_x \pi x}{a} \cdot \sin \frac{n_y \pi y}{b} \cdot \sin \frac{n_z \pi z}{c} \quad \text{— (15)}$$

Where $k = B_1 B_2 B_3 = \text{constant}$ — (16)

$$|\psi|^2 = k^2 \iiint \psi^* \psi \, dx \, dy \, dz = 1$$

From equ (15),

$$|\psi|^2 = k^2 \sin^2 \frac{n_x \pi_x}{a} \cdot \sin^2 \frac{n_y \pi_y}{b} \cdot \sin^2 \frac{n_z \pi_z}{c}$$

$$|\psi|^2 = k^2 \sin^2 \int_{-\infty}^{-1} \frac{n_x \pi_x}{a} \cdot \sin^2 \int_{-\infty}^{+\infty} \frac{n_y \pi_y}{b} \sin^2 \int_{-\infty}^{+\infty} \frac{n_z \pi_z}{c}$$

—

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$$K^2 \int_0^a \int_a^b \int_b^c \sin^2 \frac{n_x \pi}{a} \cdot \sin^2 \frac{n_y \pi}{b} \cdot \sin^2 \frac{n_z \pi}{c} \, dx\,dy\,dz = 1 \quad \text{— (17)}$$

$$\Downarrow$$

$$\because \left[ H^2 = \iiint \psi^* \psi \, dv = 1 \right] \Rightarrow \left[ \int_0^a \sin^2 \frac{n_x \pi x}{a} dx = 1 \right] = \frac{a}{2}$$

$$\Rightarrow \left[ \int_a^b \sin^2 \frac{n_y \pi y}{b} dy = 1 \right] = \frac{b}{2}$$

$$\Rightarrow \left[ \int_b^c \sin^2 \frac{n_z \pi z}{c} dz = 1 \right] = \frac{c}{2}$$

$$K^2 \times \frac{abc}{8} = 1 \quad \text{or} \quad \left[ k = \frac{2\sqrt{2}}{\sqrt{abc}} \right] \quad \text{— (18)}$$

$$\therefore \psi = k \sin \frac{n_x \pi}{a} \sin \frac{n_y \pi}{b} \cdot \sin \frac{n_z \pi}{c}$$

$$\Rightarrow \left[ \psi = \frac{2\sqrt{2}}{\sqrt{abc}} \sin \frac{n_x \pi x}{a} \sin \frac{n_y \pi y}{b} \cdot \sin \frac{n_z \pi z}{c} \right] \quad \text{— (19)}$$

$$\Downarrow$$

(Eigen Function)

Now,

$$\because \left[ \alpha_1^2 + \alpha_2^2 + \alpha_3^2 = \frac{2mE}{\hbar^2} \right]$$

$$\therefore \left[ E = \frac{\hbar^2 \pi^2}{2m} \left( \frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2} \right) \right] \quad \text{— (20)}$$

$$\Downarrow$$

Eigen value

Eigen function for 3-D box

$$\because \int_0^a \sin^2 \frac{n_x \pi x}{a} dx = 1 \quad \left| \frac{2n_x \pi x}{a} = t \right.$$

$$\frac{2n_x \pi}{a} dx = dt$$

$$dx = \frac{a}{2n_x \pi} dt$$

$$x = a \Rightarrow t = 2n_x \pi$$

$$= \frac{1}{2} \int_0^a dx - \int_0^{2n_x \pi} \cos t \cdot \frac{a}{2n_x \pi} dt$$

$$= \frac{1}{2} [x]_0^a - [\sin t]_0^{2n_x \pi} \cdot \frac{a}{2n_x \pi}$$

$$= \frac{1}{2} \int_0^a 2 \sin^2 \frac{n_x \pi x}{a} dx = \frac{1}{2} \int_0^a \left(1 - \cos \frac{2n_x \pi x}{a}\right) dx$$

$$= \frac{a}{2} - [\sin 2n_x \pi - \sin 0] \cdot \frac{a}{2n_x \pi}$$

$$= \frac{a}{2}.$$

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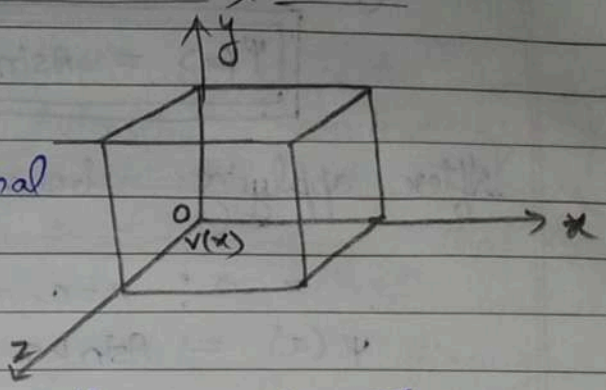
# Particles in 1-Dimensional box :-




Let us consider a particle of mass, 'm' in an one-dimensional box.

The particle is restricted to move in a straight line along the x-axis. The range of the particle is $\boxed{0 < x < a}$

The potential function V(x) has constant value ('0') in the following region.

i.e.,

$$V(x) = 0 \quad ; \quad 0 < x < a.$$

So, the potential according to Schrodinger wave equation.

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{— (i)}$$

Inside the potential $V = 0$

From equation (i), $\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}E\psi = 0}$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2mE\psi}{\hbar^2} = 0 \longrightarrow \because \hbar = \frac{h}{2\pi} \therefore \frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\left(\frac{h}{2\pi}\right)^2}E\psi = 0$$

$$\boxed{\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 mE\psi}{h^2} = 0} \quad \text{— (2)}$$

Taking $\boxed{\frac{8\pi^2 mE}{h^2} = k^2}$ — (2-a)

$$\boxed{\frac{\partial^2\psi}{\partial x^2} + k^2\psi = 0} \quad \text{— (3)}$$

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The general soln of equ (3) is :

$$\psi(x) = A\sin kx + B\cos kx \quad \text{---(4)}$$

After applying boundary condition,

$\therefore x = 0 ; \; x = a$

$\therefore \psi(a) = A\sin ka = 0$

$\Rightarrow \sin ka = n\pi \Rightarrow k = \frac{n\pi}{a}$

$$\psi_n(x) = A\sin\left(\frac{n\pi}{a}\right)x \quad \text{---(5)}$$

$$E = \frac{k^2h^2}{8\pi^2 m} \quad \text{---(6)}$$

$\Rightarrow$ Eigen Value for 1-D box

After applying Normalization condition;

$$\int_0^a |\psi|^2 dx = 1$$

$$\Rightarrow A^2 \int_0^a \sin^2\left(\frac{n\pi x}{a}\right) dx = 1$$

$$\Rightarrow A^2 \int_0^a \frac{\left(1 - \cos 2\left[\frac{n\pi x}{a}\right]\right)}{2} dx = 1$$

$$\Rightarrow \frac{A^2}{2}\int_0^a \left\{1 - \cos 2\left(\frac{n\pi x}{a}\right)\right\} dx = 1.$$

$$\Rightarrow \frac{A^2}{2} a = 1 \Rightarrow A = \sqrt{\frac{2}{a}}$$

$$\psi_x = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

$\Rightarrow$ Eigen function in 1-Dimensional

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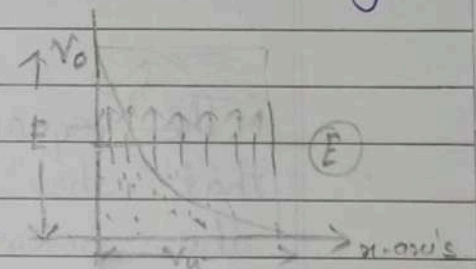
21/10/23

# Tunnel Effect

Consider a particle with a definite energy E moving towards a potential barrier of ht $(V_0 > E)$. According to classical mechanics, the particles can't go through that barrier, but in quantum mechanics, there is not a null probability that the particle goes through the barrier.

This is a consequence of 1st Schrodinger Equo.

In the following, we will study the tunnel effect w.r.t. a particle moving in only in one direction!




According to quantum mechanics,

the probability that the particle penetrate the potential barrier is not null.

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi(x) = 0 \quad \text{—— (i)}$$

Equo (i) is linear differential equo with constant coefficient.

The auxiliary equo is :—

$$k^2 + \frac{2m}{\hbar^2}(E-V) = 0 \text{ has two soln.}$$

$$\Rightarrow \boxed{k_1 = +\sqrt{\frac{2m}{\hbar^2}(E-V)}.}$$

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& $\boxed{k_2 = -\sqrt{\frac{2m}{\hbar^2}(E-V)}}$

Thus, the two sol$^n$ of equ (i) are the real functions.

$\boxed{\psi_1(x) = e^{xk}}$ & $\boxed{\psi_2(x) = e^{-xk}}$,

where [, e is the base of natural logarithm]

Since, the 1st divergence solution is not acceptable. The general sol$^n$ for equ (i) is real function.

Therefore, $\boxed{\psi(x) = Ae^{-xk}}$ → Eigen function.

The square of its moduk gives the probability that the particle is in a point 'x' occupied by the potential barrier.

$$p(x) = |A \cdot e^{-xk}|^2.$$

$$\Rightarrow \boxed{p(x) = A^2 \cdot e^{-2xk}}$$

The probability that the particle is in a point inside the potential barrier is not null.

According to Schrodinger Quantum mechanics, a particle can penetrate in a potential barrier ~~is~~ in a ~~mental~~ matter prohibited by a classical mechanics

Project :- * Polarization of light
* Plane, circularly & elliptically
* Brewster law
* Malus law
* Polarized ligh.
* double refl.
* Nicol Prism.

Date
Page

Definition
figure
Application

11/01/24

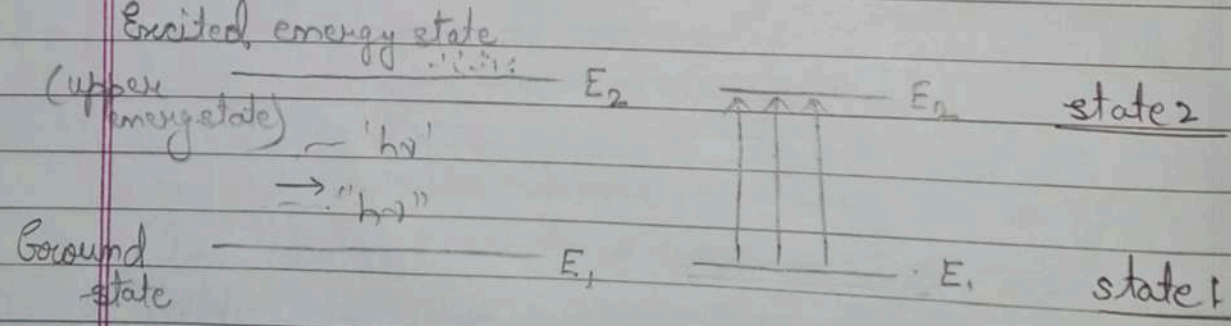
# LASER

LASER :- Light Amplification Stimulated Emission of Radiation.

* * Absorption of radiation
* Spontaneous emission
* Stimulated emission
* Population Inversion.
* Normal Population.

## # Absorption of radiation :-

At low temp., most of the atoms stay in lower energy state. if an atom is ~~initial~~ initially in the lower energy state $E_1$. It can be raised to the higher energy state $E_2$ by the absorption of photon of energy ($h\nu$), this is known as absorption of radiation.




(Lower energy state)

(Before absorption)

$$E_2 - E_1 = h\nu$$

$$\Delta E = h\nu \quad \leftarrow (i)$$

The probability of occurance of this absorption from state 1 to state (2) is proportional to the energy density $u(\nu)$ of the radiation.

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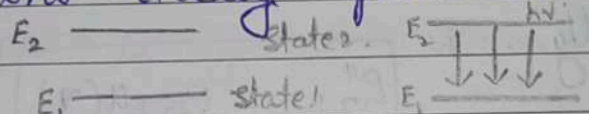
$\therefore \delta E = P_{12} = B_{12}$ — (2)

where $B_{12}$ is known as Einstein coefficient of absorption of radiation.

# Spontaneous Emission :-

(i) If an atom is initially in the upper energy state $E_2$, it can come down to the lower energy state $E_1$ by emitting the photon of energy $h\nu$. (ii) This is known as spontaneous radiation. (ii) This is the natural radiation decay process, i.e., inherent in all excited states of all materials.

However, such emission is not always the dominant decay process.




(iii) The probability of occurance of this absorption spontaneous emission, transient from state 2 to state 1 depends only on the properties of state 1 & 2.

$$P'_{21} = A_{21} \quad \text{— (3)}$$

where $A_{21}$ is known as Einstein coefficient of emission of radiation.

# Stimulated Emission :- (Inherent Emission).

(i) Einstein ~~works~~ was the first to point out the 3rd possibility of induced emission in which an incident photon of energy $h\nu$ causes the transition from upper to lower energy state.

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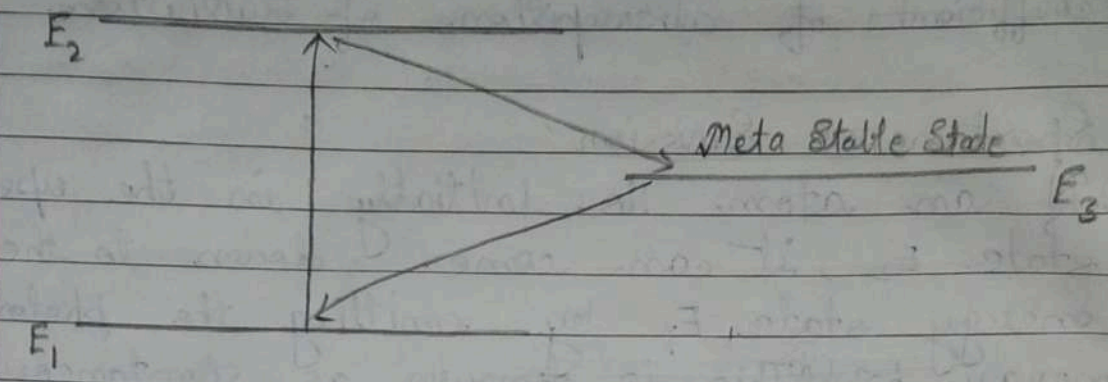
# LASER




(ii) The probability of occurance of stimulated emission transition from state 2 to state 1 is proportional to energy density of radiation.

Finally, $$P''_{21} = B_{21} u(\nu) \quad \text{— (4)}$$

where, $B_{21}$ is known as stimulated emission of radiation.

$$\therefore P_{21} = P'_{21} + P''_{21}$$

$$\Rightarrow P_{21} = A_{21} + B_{21} u(\nu) \quad \text{— (5)}$$

VVI

Q. Population Inversion is the necessary criteria for laser action. Why?

⇒ Let a system in which $N_1$ be the no. of atoms are there on the ground state.
And let $N_2$ be the no. of atoms are there on the excited state.
For amplification of light to occur the condition is

$$N_2/N_1 > 1$$

$$\Rightarrow N_2/N_1 = e^{(E_2-E_1)/kT} = e^{h\nu/kT}$$

That's why, this is the necessary condition for laser action. Ans

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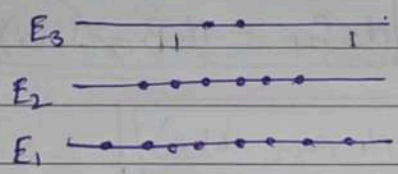
# Normal Population :-

In thermal equilibrium, when the no. of atoms in ground state is generally much greater than in a higher energy state. This is known as normal population of atoms.

It is represented as $N_1 > N_2 > N_3$.

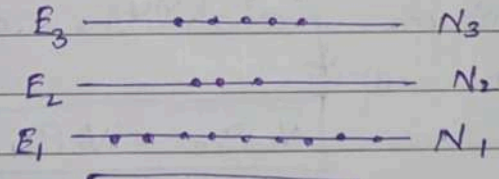



$N_1 > N_2 > N_3$

# Population Inversion :-

A state in which the no. of atoms in higher energy state is greater than that of lower energy state, is known as population inversion.

It is represented as




$\boxed{N_1 > N_3 > N_2}$

# Properties of laser :-

(i) Coherent
(ii) Monochromatic
(iii) Collimated

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01/02/24

VVI

# Relation b/w Einstein A & B coefficients

Let $N_1$ & $N_2$ be no. of atoms at any instant in state (1) & state 2 resp. The

The probability of absorption transition for no. of atoms from state 1 to state 2 per unit time is given by

$$N_1 P_{12} = N_1 B_{12} U(\nu) \quad \text{---} (6)$$

$$N_1 P_{12} = N_1 B_{12} u(\nu) \quad \text{---} (6)$$

The total probability of transition for no. of atoms from state 2 to 1 either by spontaneously or by stimulated emission per unit time is given by

$$N_2 P_{21} = N_2 [A_{21} + B_{21} u(\nu)] \quad \text{---} (7)$$

In thermal equilibrium at temp. T, the absorption and emission probabilities are equal and thus, we can write

$$N_1 P_{12} = N_2 P_{21}$$

or $N_1 B_{12} u(\nu) = N_2 [A_{21} + B_{21} u(\nu)]$

$\Rightarrow N_1 B_{12} u(\nu) = N_2 A_{21} + N_2 B_{21} u(\nu)$

$\Rightarrow u(\nu) [N_1 B_{12} - N_2 B_{21}] = N_2 A_{21}$

$$\Rightarrow u(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}}$$

$$\Rightarrow u(\nu) = \frac{A_{21}}{B_{21}\left[\left(\frac{N_1}{N_2}\right)\left(\frac{B_{12}}{B_{21}}\right) - 1\right]} \quad \text{---} (8)$$

[By dividing num. & deno. by $(N_2 B_{21})$]

But a/c to Einstein,

$$B_{12} = B_{21} \quad \text{---} (9)$$

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So, from equo (8) & (9), $u(\nu) = \frac{A_{21} \times \frac{1}{(B_{21}) \times \left(\frac{N_1}{N_2}\right) - 1}}{}$ — (10)

A/c to Boltzmann's law, the distribution of atoms among the energy states $E_1$ & $E_2$ at the thermal equilibrium at temp. T is given by

$$\frac{N_1}{N_2} = \frac{e^{-E_1/kT}}{e^{-E_2/kT}} = e^{(E_2 - E_1)/kT} \quad \text{— (11)}$$

$$\Rightarrow \boxed{N_1/N_2 = e^{h\nu/kT}} \quad \text{— (12)}$$

Here, k = Boltzmann constant

Then, So, equo (10) can be written as

$$\boxed{u(\nu) = \frac{A_{21} \times \frac{1}{(B_{21}) \times (e^{h\nu/kT}) - 1}}} \quad \text{— (13)}$$

But, a/c to Plank's radiation formula,

$$\boxed{u(\nu) = \frac{8\pi h\nu^3}{c^3} \times \frac{1}{(e^{h\nu/kT}) - 1}} \quad \text{— (14)}$$

Comparing equo (13) & (14), we get

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}} \quad \text{— (15)}$$

Equo (15) gives the relation b/w the probabilities of spontaneous & stimulated emission.
This is also known as the relation b/w the Einstein's coefficients A & B.

# Pumping :-

The process of supplying energy to excited atoms from their lower energy state to higher energy state, is called pumping or optical pumping.

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There are two types of pumping :-

(i) Direct Pumping
- (a) Optical Pumping
- (b) Particle Pumping

(ii) Indirect Pumping
- (a) 2-level laser.
- (b) 3-level laser → Ruby laser
- (c) 4-level laser → He-Ne laser

05/02/24

Ruby laser.
- Construction
- Working.
- Energy level diagram
- Efficiency.
- Application

Ruby rod → ($Al_2O_3$ crystal)

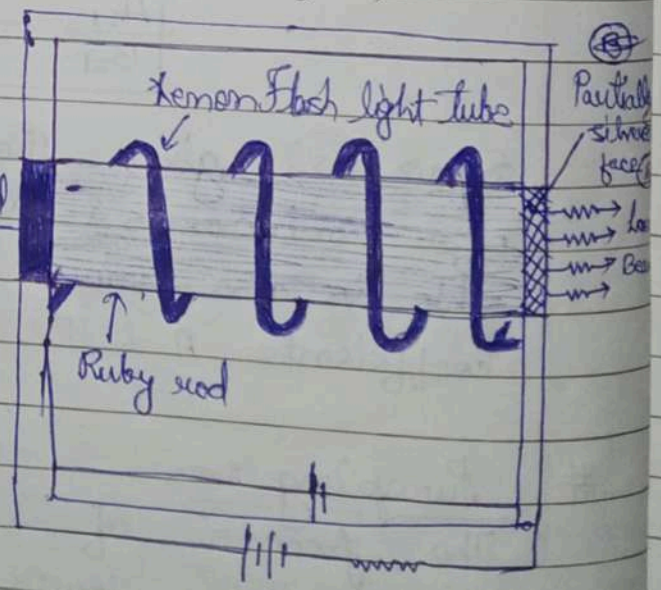
Resonator (optical cavity)

# Ruby laser :- Ruby laser is a solid state laser, which consists of 3 main parts (i) working material (ii) Optical resonant cavity (iii) excitation source.

Construction of Ruby laser :-

Ruby laser is made up of cylindrical ruby crystal of length 2 cm to 30 cm in length and 0.5 cm to 2 cm in diameter. These rods are surrounded by liquid nitrogen to maintain their temperature. When high temp. is produced during the operation

Fig (a)

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of the laser. One of the end is fully silvered and other is partially silvered, so that they can act as fully & partially reflected surfaces.

- Pumping Source:- This is required to create population inversion in ruby laser and generally a helical flash lamp filled with Xenon, is used for this purpose.

# Working :-

In the normal state, most of the chromium ions are in the ground state ($E_1$). When light from the flash tube is of wavelength (5500 Å) is made to fall upon the ruby rod; these incident photons are observed by the chromium ions that rise to the excited state ($E_3$). Then, they give a part of their energy to the crystal structure and reach the metastable state ($E_2$). These ions are in the meta-stable state can remain for a longer duration $10^{-3}$ sec. Therefore, the no. of ions in this state goes on increasing while at the same time no. of ions in ground state goes on decreasing due to the optical pumping.

Thus, the population inversion is established b/w the metastable state and the ground state. When an excited ions passes spontaneously from meta stable state to the ground state, it emits a photon of energy 6000 Å.

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# Energy level diagram :-

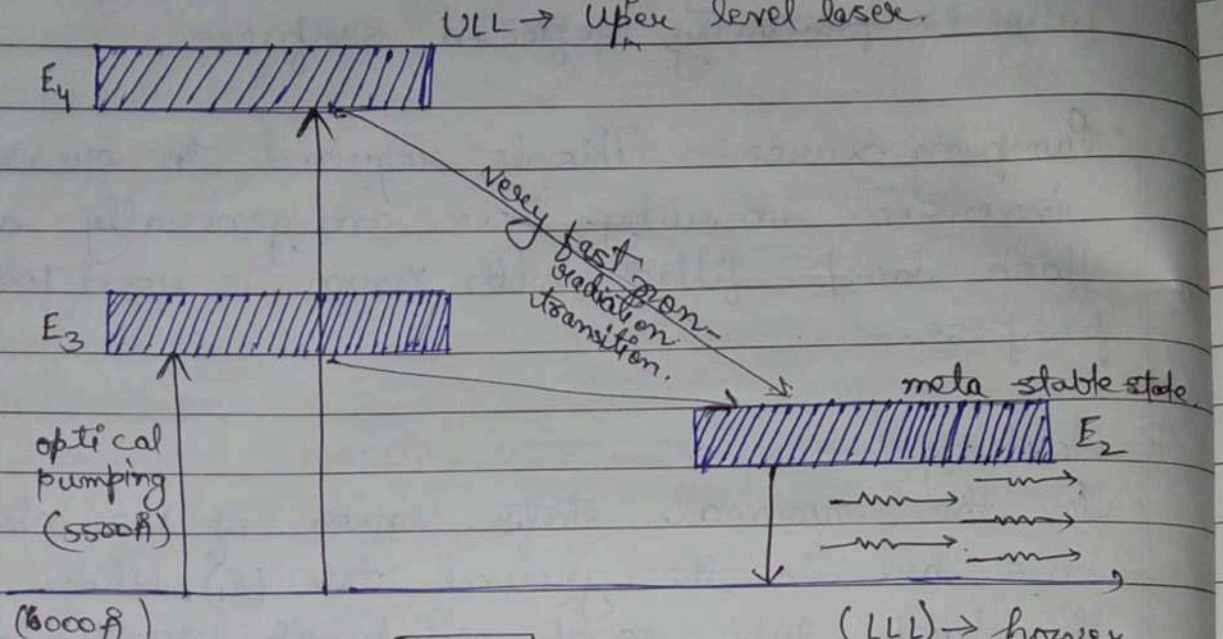


Fig. B

(Energy level diagram of $Cr^{3+}$ ions in Ruby laser)

The output power of ruby laser operates at 1% efficiency. It may produce the laser beam of 1 mm to 25mm in diameter.

# Application :-

Ruby laser is used for
(i) Cutting (ii) welding (iii) holography.

# Disadvantage :-

(i) It requires high pumping power.
(ii) The efficiency of laser is low as it is difficult to maintain the population inversion.
(iii) The output is not continuous but occurs in the form of pulses of microsecond duration.

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9/02/24

# He-Ne laser / four level laser

Construction :-

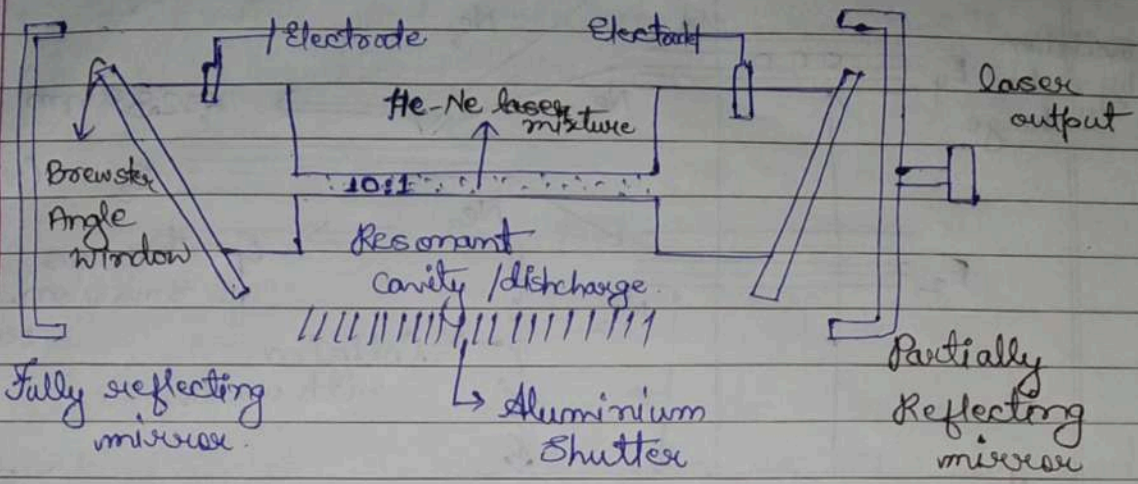


Fig.(a) He-Ne laser construction

→ The gases are taken along cylindrical discharge tube, He-Ne (Helium & Neon) mixture in the ratio (10:1) is taken as the active medium and neon acts as the active centre.

→ There are two mirror, one fully reflecting and other partially reflecting & partially transmitting to couple out the laser b/w.

→ The gases pressure is maintained as 2.5 torr and the gas temp. is maintained as 400 K. arrangement for polarized laser output is produced by Brewster's angle window in the laser cavity.

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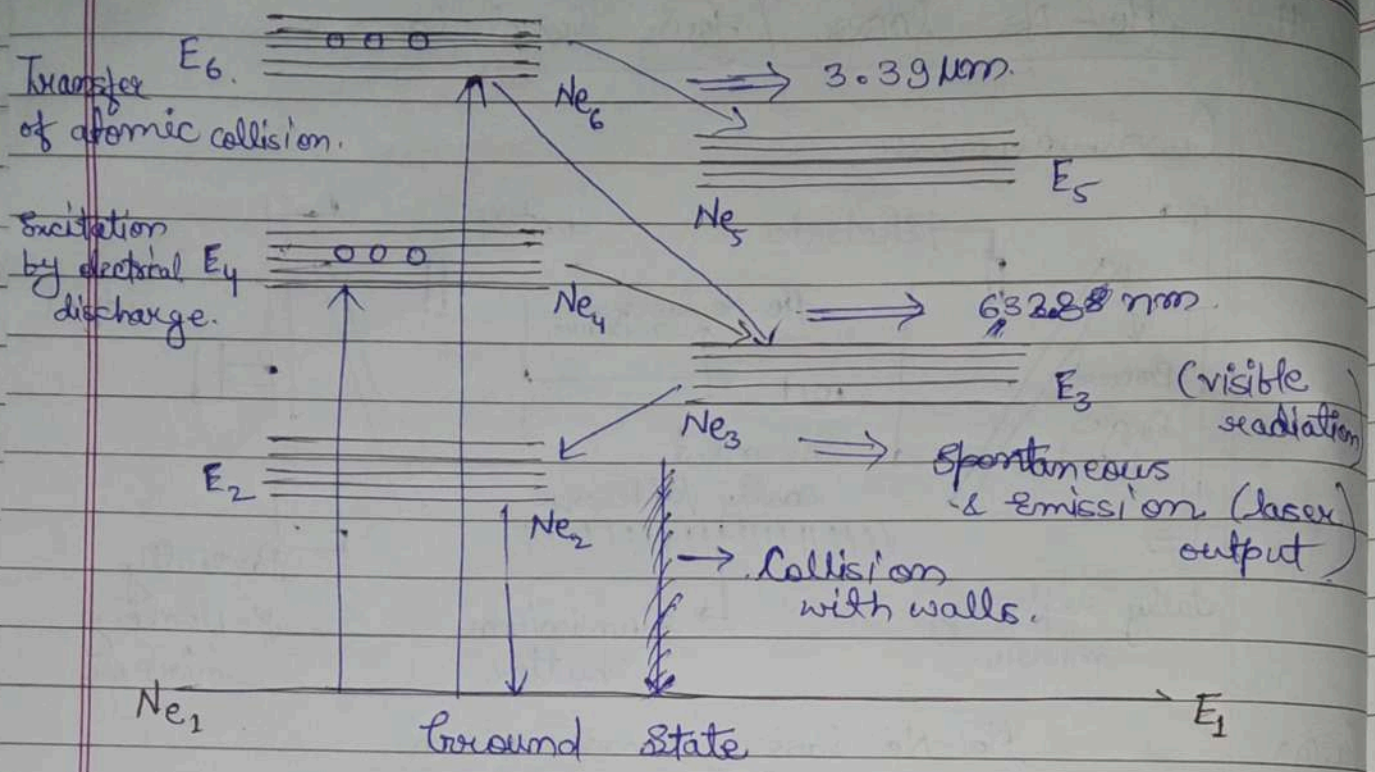


Fig.(b) :- Energy level diagram of He-Ne laser

⇒ The following transition takes place :—

$Ne_1 \longrightarrow Ne_4 \longrightarrow$ Excitation

$Ne_1 \longrightarrow Ne_6 \longrightarrow$ Excitation

$Ne_6 \longrightarrow Ne_5 \longrightarrow$ 3.39 μm (Infrared radiation

$Ne_6 \longrightarrow Ne_3 \longrightarrow$ 6328 nm (visible radiation)

$Ne_4 \longrightarrow Ne_3 \longrightarrow$ 1.15 μm (Infrared)

$Ne_3 \longrightarrow Ne_2 \longrightarrow$ Orange

$Ne_2 \longrightarrow Ne_1 \longrightarrow$ Collision

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# Output Characteristics :-

The He-Ne laser has the standard wavelength of 6328 Å and other visible wavelength can also be produced. like.

5430 Å → Green
5940 Å → Yellow
6120 Å → Red & Orange.

The He-Ne laser operates in continuous working mode & overall gain is very low and it is typically about 0.01% to 0.1 % observed. The laser is simple, practical & less expensive. The laser beam is highly polluted, coherent and monochromatic.

# Application

It is used for

(i) laser printing
(ii) bar code reading
(iii) fibre optic communication.
(iv) Interferometer
(v) laser scanner
(vi) Alignment in pipe layering in laboratories

Date
Page

## Relation b/w Group velocity & Particle velocity

Consider a material particle of rest mass $m_0$. Let its mass be $m$ when it moves with velocity '$v$'. Then its total energy is given by

$$E = mc^2$$

$$\Rightarrow \boxed{E = m_0c^2/\sqrt{1-v^2/c^2}} \quad \text{---(1)}$$

Its momentum is given by $\boxed{p = mv = \frac{m_0v}{\sqrt{1-v^2/c^2}}}$ ---(2)

We know that,

the relation b/w E & p of relativistic particle is

$$E = \sqrt{p^2c^2 + m_0^2c^4} \Rightarrow E^2 = p^2c^2 + m_0^2c^4.$$

on diff., we get $2E\,dE = 2p\,dp \cdot c^2 + 0$.

$$\Rightarrow \boxed{\frac{dE}{dp} = pc^2/E} \quad \text{---(3)}$$

Al

12/02/24

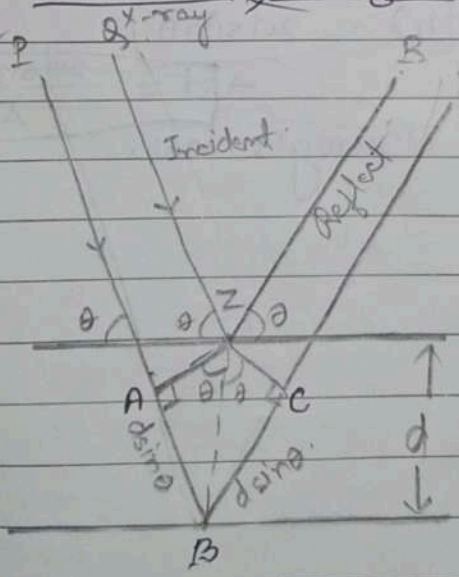
# Modern Physics

* Braag's law :-
* Compton effect
* Photo electric effect.
* Heisenberg Principle

# Braag's law :- When monochromatic x-ray impinged upon the atom in a crystal ## lattice acts as a series of || reflecting planes and each atom acts as a source of scattering radiation of the same wavelength.

The intensity of the reflected beams at certain angle will be maximum when the path difference b/w the 2 reflected waves from 2 different planes is an integral multiple of wavelength 'λ'.

# Derivation of Braag's law :-




AB + BC = path difference

$$n\lambda = 2d\sin\theta$$

d → Crystal lattice (spacing b/w the atomic plane)

Fig :- Braag's law

Let us ~~emarin~~ emagine a beam of x-ray entering a crystal with one of these plane of atoms oriented at an angle 'θ' to the incoming beam of monochromatic x-ray.

(i) From fig., if the path difference is an integer multiple of $n\lambda$, then the rays will be in phase when they exist from the crystal constructive interference occur.

(ii) If the path difference is not an integral multiple of wavelength, then destructive interference will occur and the wave will not as strong as when they enter the crystal

⇒ The condition for constructive interference is $\boxed{n\lambda = AB + BC}$

From fig, we know that, $AB = d\sin\theta$

$AB + BC = 2d\sin\theta$

$\Rightarrow \boxed{n\lambda = 2d\sin\theta}$

$\Rightarrow \boxed{d = \frac{n\lambda}{2\sin\theta}}$

$\Rightarrow \boxed{d = \frac{2d\sin\theta}{\lambda}}$

This is known as Braag's law.

# Photoelectric Effect :-

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VVI

# Compton Effect :- (Particle nature of electromagnetic wave)

The phenomenon of increase in the wavelength (or decrease in the frequency) of X-ray radiation by scattering, is called the Compton Effect.

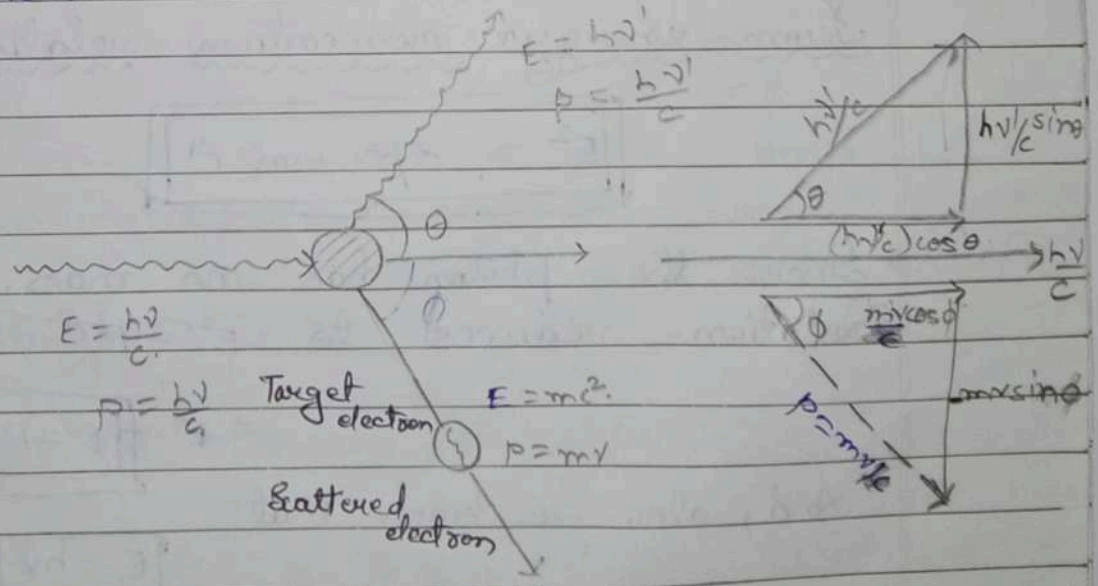
With the success of Quantum theory of radiation, energy is considered to be transferred in the form of "Quanta." or tiny bundles of packets. oscillating with frequency "$\nu$". These tiny packets of energy are called "photons." The energy of the photon is given as $\boxed{E = h\nu}$

where, $h$ = plank's constant.

$\boxed{h = 6.626 \times 10^{-34}\ Js}$

Compton Quantum effect is one such phenomena which confirms the particle nature of electromagnetic radiation.

Consider a photon of energy $h\nu$ that collides with an electron is initially at rest, mass energy of "$m_0c^2$". After collison, the photon gets scattered and moves in a direction inclined at an angle $\theta$ with the initial direction of photon as shown in figure.




During collision, the photon looses some amount photon of energy to the electron i.e., $(h\nu')$. At same

time, the electron gains energy and moves with velocity 'v' at an angle $\phi$.
Let the final energy of electron be $(mc^2)$.

The loss in energy of the photon is equal to gain of energy of the electron. And this is equal to kinetic energy of the electron.

$$\therefore \boxed{mc^2 - m_0c^2 = \frac{1}{2}mv^2} \quad \text{---(1)}.$$

A/c to the principle of conservation of total energy before collision must be equal to total energy after collision.

i.e., $h\nu + m_0c^2 = h\nu' + mc^2$.

$$\Rightarrow \boxed{mc^2 = m_0c^2 + h(\nu - \nu')} \quad \text{---(2)}$$

From energy momentum relation;

$$\boxed{E^2 = c^2p^2 + m_0^2c^4}$$

Since the photon has no mass, the above relation reduces to $E^2 = p^2c^2$.

$$\Rightarrow \boxed{E = pc} \quad \text{---(3)}$$

And, also we know that

$$\boxed{E = h\nu} \quad \text{---(4)}$$

From equn (3) & (4),

$$\boxed{p = \frac{h\nu}{c}} \quad \text{---(5)}$$

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From the principle of conservation of momentum, the total momentum before collision must be equal to the total momentum after collison in each of the two is mutually perpendicular direction.

In the initial photon direction;

on x-axis :-

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\theta + \frac{mv\cos\phi}{\sqrt{}}$$

$$\Rightarrow mvc\cos\phi + h\nu'\cos\theta = h\nu$$

$$\Rightarrow \boxed{h(\nu - \nu'\cos\theta) = mvc\cos\phi} \quad \text{--- (A)}$$

Squaring equo (A) both sides, we get

$$\boxed{m^2v^2c^2\cos^2\phi = h^2(\nu^2 + \nu'^2\cos^2\theta - 2\nu\nu'\cos\theta)} \quad \text{--- (6)}$$

Again on y-axis

Before Collision = After collision

$$\Rightarrow 0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi$$

$$\Rightarrow \frac{h\nu'}{c}\sin\theta = mv$$

$$\Rightarrow \frac{h\nu'}{c}\sin\theta = mv\sin\phi$$

$$\Rightarrow h\nu'\sin\theta = mvc\sin\phi \quad \text{--- (B)}$$

On squaring equo (B) both sides, we get.

$$h^2\nu'^2\sin^2\theta = m^2v^2c^2\sin^2\phi \quad \text{--- (7)}$$

Adding (6) & (7), we get

$$m^2v^2c^2(\cos^2\phi + \sin^2\phi) = h^2(\nu^2 + \nu'^2\cos^2\theta - 2\nu\nu'\cos\theta) + h^2\nu'^2\sin^2\theta$$

$$\Rightarrow m^2v^2c^2 = h^2\nu^2 + h^2\nu'^2\cos^2\theta - 2h^2\nu\nu'\cos\theta + h^2\nu'^2\sin^2\theta$$

$$\Rightarrow \boxed{m^2v^2c^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\theta} \quad \text{--- (8)}$$

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(Equo 2)$^2$ − equo (8), we get

$\Rightarrow m^2c^4 - m^2v^2c^2 = m_0^2c^4 + h^2(\nu-\nu')^2 + 2h(\nu-\nu')m_0c^2 - h^2\nu - h^2\nu' + 2h^2\nu\nu'\cos\theta$

$\Rightarrow m^2c^4\left(1-\frac{v^2}{c^2}\right) = m_0^2c^4 - 2h^2\nu\nu' + 2h(\nu-\nu')m_0c^2 + 2h^2\nu\nu'\cos\theta$

$\Rightarrow \boxed{m^2c^4\left(1-\frac{v^2}{c^2}\right) = -2h^2\nu\nu'(1-\cos\theta) + m_0^2c^4 + 2h(\nu-\nu')m_0c^2}$ ——— (9)

Now, by relativistic formula,

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

$$\Rightarrow m^2 = \frac{m_0^2}{1-v^2/c^2} \Rightarrow \boxed{m^2\left(1-\frac{v^2}{c^2}\right) = m_0^2}$$ ——— (10)

From equo (9) & (10),

$m_0^2c^4(1-v^2) = -2h^2\nu\nu'(1-\cos\theta) + m_0^2c^4 + 2h(\nu-\nu')m_0c^2$

$\Rightarrow 2h^2\nu\nu'(1-\cos\theta) = 2h(\nu-\nu')m_0c^2$

$\Rightarrow \boxed{\frac{h}{m_0c^2}(1-\cos\theta) = \frac{\nu-\nu'}{\nu\nu'}}$ ——— (C)

Also, we know that,

$\lambda = \frac{c}{\nu} \Rightarrow \lambda' = \frac{c}{\nu'}$

$\Rightarrow \Delta\lambda = \lambda' - \lambda$

$= \frac{c}{\nu'} - \frac{c}{\nu} = c\left[\frac{1}{\nu'} - \frac{1}{\nu}\right]$

$\Rightarrow \frac{\lambda'-\lambda}{c} = \left[\frac{\nu-\nu'}{\nu\nu'}\right]$ ——— (D)

From equo (C) & (D),

$\boxed{\Delta\lambda = \frac{h}{m_0c}(1-\cos\theta)}$ proved

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# Heisenberg's Uncertainty Principle :-

→ It is perhaps the best known result of the wave particle's duality. A/c to Heisenberg's uncertainty principle, it is impossible to determine simultaneously the exact position and momentum of a small moving particle like electron.

→ It can be proved on the basis of de Broglie wave concept that the material particle is equivalent to a group of waves, as group velocity (G) is equal to the particle velocity (v).

→ Consider a simple case of group of waves which is formed by the superposition of two harmonic plane (simple) waves of equal amplitudes a & having nearly equal frequencies $\omega_1$ & $\omega_2$. So, the two waves can be represented by

$$\boxed{Y_1 = a\sin(\omega_1 t - k_1 x)}$$
$$\& \quad \boxed{Y_2 = a\sin(\omega_2 t - k_2 x)}$$

where,

$k_1$ & $k_2$ are their propagation constant & $\omega_1/k_1$ & $\omega_2/k_2$ are phase velocities. The resultant wave due to the superposition of two waves is given by

$$Y = Y_1 + Y_2$$
$$\Rightarrow \boxed{Y = 2a\sin(\omega t - kx)\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right]} \quad \text{— (i)}$$

where, $\omega = \frac{\omega_1+\omega_2}{2}$, $k = \frac{k_1+k_2}{2}$ & $\Delta\omega = \omega_1 - \omega_2$ & $\Delta k = k_1 - k_2$

The resultant wave is shown in fig. (A). The loop of this wave travels with the group velocity (G), given by

$$G = \frac{\Delta\omega}{\Delta k} = \frac{\omega_1 - \omega_2}{k_1 - k_2}$$

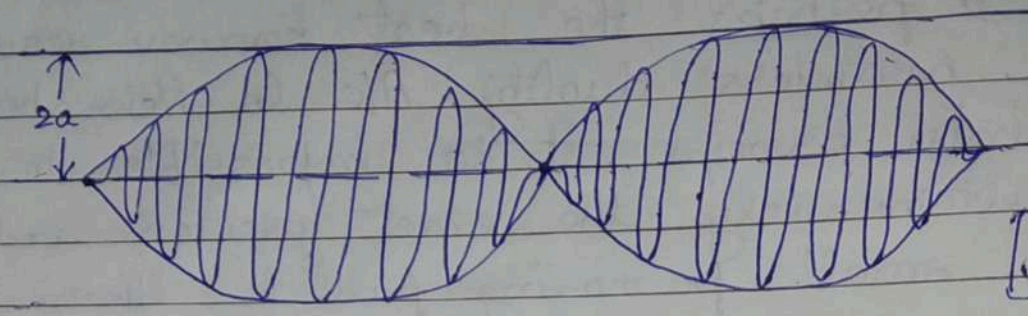


Fig A

Since, the group velocity is equal to the particle velocity. Then the particle may be anywhere within the loop. Now, the condition of the formation of node from equ (i) is given by

$$\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right] = 0$$

$$\Rightarrow \frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x = \frac{\pi}{2}, \frac{3\pi}{2}, -, \frac{(2n+1)\pi}{2} \quad \text{(ii)}$$

where, $n = 0, 1, 2, --$

If $x_1$ & $x_2$ are the values of the position of two consecutive nodes, then from above equ by putting 'n' & (n+1) resp., we get.

$$\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x_1 = \frac{(2n+1)\pi}{2} \quad \text{(iii)}$$

$$\& \quad \frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x_2 = \frac{(2n+3)\pi}{2} \quad \text{(iv)}$$

(iv) — (iii),

$$\boxed{\frac{\Delta k}{2}(x_1 - x_2) = \pi}$$

$$\Rightarrow \boxed{\frac{\Delta k}{2}\Delta x = \pi} \quad \text{(v)}$$

But, also, $k = \frac{2\pi}{\lambda} \Rightarrow = \frac{2\pi}{h/p} = \frac{2\pi p}{h}$

$$\Rightarrow \boxed{\Delta k = \frac{2\pi}{h}\Delta p} \quad \text{(vi)}$$

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From equo (v) & (vi), $\Delta x = \frac{h}{\Delta p} \Rightarrow \boxed{\Delta x \cdot \Delta p = h}$

However, the more accurate measurements show that the product of uncertainties in momentum ($\Delta p$) and the position ($\Delta x$) can't be less than $h/2\pi$. Therefore,

$$\boxed{\Delta x \cdot \Delta p \geq \hbar}$$

This is Heisenberg's Uncertainty principle.

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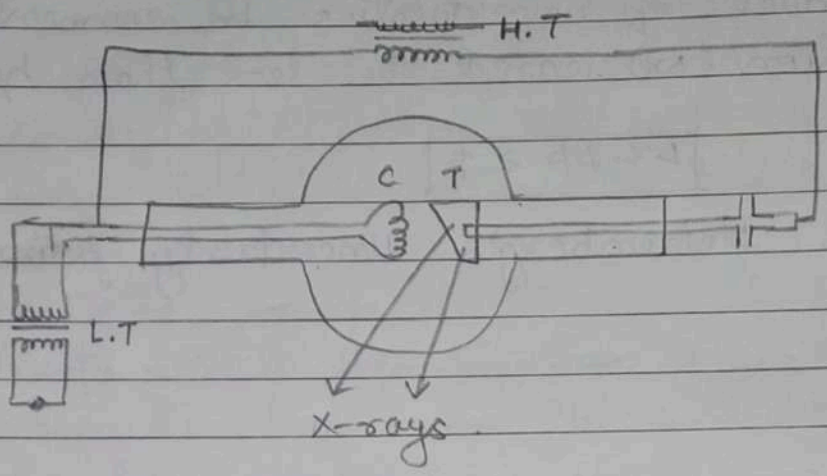
# CH-7
## X-rays

X-rays is an Electromagnetic wave having high penetrating power.




- X-rays are produced when high melting pot fast moving electrons strike with a target of high melting point and high atomic no. Ex:- tungsten, copper etc.
- A coolidge tube is generally used to produce the x-rays as shown in the figure.
- It is evacuated at low pressure of $10^{-5}$ mm of Hg. This tube contains a cathode C and a target T.
- The filament is heated with a low voltage source to emit electrons and high voltage source b/w filament & target to accelerate electrons.
- When these high velocity electrons strike the target, x-rays are produced.
- Only about 0.2% of electron beam energy is used to produce x-rays, the rest of the energy transforms into heat.

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- To remove heat, target material is mounted on a hollow copper tube through which the cold water is flowing continuously.

Control of Intensity :-

- The intensity of x-rays is controlled by controlling the intensity of the incident electrons striking the target.
- The intensity of x-rays is controlled by controlling the current in the filament.

Control of Penetrating Power :-

- The penetrating power of x-rays is controlled by controlling the potential diff. b/w the filament and the target.

# Properties (or Characteristics) of x-rays :-

- X-rays are electromagnetic radiations of very short wavelength of the order $10^{-10}$ m.
- They travel with the velocity of light.
- The x-rays are not deflected in the presence of electric & magnetic fields.
- They produce fluorescence in some substances like barium platino-cyanide, zinc sulphide etc.
- They effect the photoelectric plate and the effect is more intense than light.
- X-rays ionize the gas when they are passing through it.
- They produce photoelectric effect.

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